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ABSTRACT

Designed to provide intermediate grade students with experience using fractions and mixed numbers, the PLATO sequence described in this manual includes interactive models for students to work with, as well as lessons for review, practice, or experience. These lessons are intended to be integrated with the classroom mathematics instruction, and introductory materials describe the implementation of the lessons, student assignments, the student session, data feedback to teachers, and supplementary materials. The development of the curriculum and results of external evaluation studies are also briefly reviewed. The descriptions of 16 selected modules that make up the bulk of the manual include a statement of purpose, brief description, and sample computer displays for each lesson. Topics covered include the meanings, ordering and equivalence, conversions, addition, subtraction, and multiplication of fractions and mixed numbers, as well as the meanings of decimal fractions and conversions to or from fractions whose denominator is a power of 10. Appendices include samples of student responses in creative activities shared with other students, a brief review of some other uses of the PLATO fractions materials, simplified flow charts illustrating the instructional modules, a list of related publications, and an index to the lesson descriptions by title.

(CHC)

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FRACTIONS CURRICULUM
OF THE
PLATO ELEMENTARY SCHOOL MATHEMATICS PROJECT
2nd Edition

St. Louis, Missouri
District Office

Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

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The PLATO Fractions Curriculum

The PLATO Fractions Curriculum covers the following

Fractions and Mixed Numbers --

- meanings
- ordering and equivalence
- conversions
- addition, subtraction, and multiplication

Decimal Fractions --

- meanings
- conversions to/from fractions whose denominator is a power of 10

The curriculum offers the student extensive experience with fractions and mixed numbers in working with carefully chosen models. The models are used interactively, not just as illustrations, and they provide mathematically relevant visual feedback in response to the student's manipulations. Lessons usually allow many different ways of solving problems. This provides the opportunity for exploration, helps the student understand the meaning of what he is doing, and helps avoid the notion that mathematics is simply the repetition of dictated algorithms.

In addition to instructional lessons, there are other types of lessons: review, practice, or experience. Most of the instructional and practice lessons are mastery-based, with the difficulty and complexity of tasks automatically adjusting (up or down) based on the student's response. The "mastery level" and the student's "current level" are indicated on each lesson's screen displays, so that the student has immediate and meaningful feedback about his progress through the lesson. The amount of time required to complete a lesson can vary widely from student to student.

Many of the lessons foster sharing of ideas among students. In addition to lessons that are designed for only two students at the same terminal, "libraries" and interterminal lessons let students share their work and ideas with others, even with students from other classes and other schools. More information about "library" lessons and some samples of student work from such lessons are presented in Appendix I.

Implementation in the Classroom

These materials were designed for use by intermediate grade students in a classroom situation where PLATO can be integrated with the classroom mathematics instruction, each reinforcing the other for a complete learning experience. (Some uses by other populations are described in Appendix II.) It is recommended that each student use PLATO daily.

In the elementary school classroom, PLATO terminals are placed two to four terminals per room, so that the teacher who is responsible for mathematics instruction is also responsible for PLATO. This encourages use of

PLATO as a resource for the teacher's math program and permits greater integration of PLATO work into the classroom mathematics program.

Terminals are placed near each other in order to encourage interaction among the students. A student will often be proud of something he has done and will be eager to show or explain it to a friend. Other time a student may turn to a classmate for help with a lesson. Generally the materials are so motivating that the interaction among students at the terminals is centered around their work and does not take them "off task". A student who needs help is rarely willing to let another student take over and do the work. Usually the student will accept only the help he needs in order to become self-sufficient again.

Because tasks are generated for each student individually, the students find that it is not helpful to copy an answer from a friend's screen. They can, however, learn ideas and strategies from watching each other, and this is regarded as an advantage.

Student Assignmer

The classroom teacher uses a terminal to assign PLATO topics for the students. Different students may be assigned different lessons, or the same lessons, or different versions of the same lessons, depending on the individual needs of the students. Lessons can be assigned in pre-arranged sets, called "modules", so that routine assignments are quick and straightforward to carry out. The teacher can also provide background information about each student's previous experience. PLATO updates this information continuously as the student progresses through the curriculum. In generating exercises, the PLATO lesson takes the student's background into account. For example, a numberline lesson on mixed numbers may include negative numbers for one student, but avoid them for another, depending on the information the teacher has given about each student's previous mathematical experience.

Using a terminal, the teacher also sets the length of student PLATO sessions. Most teachers tend to allow about 30 minutes for each student's daily PLATO session, except on short days when assemblies, field trips, or other activities necessitate setting a shorter time. An automatic "management system", which is a fundamental part of the curriculum package, takes care of translating the teacher's assignments and other decisions into personalized instructional sessions for the students.

The Student Session

Following is one structure which is typical of what students encounter when they sit down at the terminal to do their day's PLATO math. After the initial sign-on procedure, the first few minutes of the session are spent in quick review exercises. These exercises are usually practice with recently learned material. The management system "remembers" the student's current performance level in each lesson, so that the difficulty of the review will always be appropriate for each student.

Next the student gets a list of several appropriate instructional lessons to choose from. (See Fig. 1.) These lessons are the main material that the student is working on. As the student masters skills and concepts presented in these lessons, new lessons are automatically added to the choice list based on the teacher's assignments, while lessons with which the student is finished disappear from the list. Appendix III outlines the sequences of lessons that have been used most often. Students can enter and leave lessons at will, but must meet completion criteria in order to obtain access to new lessons by finishing old ones.

Near the end of the session, the list of available lessons that the student can choose to work on is expanded to include several general experience lessons. (See Fig. 2.) Some of these lessons are games that reinforce recently learned mathematical skills. Others provide further practice in previously learned skills or encourage initial exploration of new areas.

When the student's time on PLATO has expired, his session is automatically ended. If he is not at a convenient stopping point, he is allowed a few minutes leeway to finish up whatever he is working on. (As with other parameters that control the student's PLATO sessions, this leeway is a number of minutes specified by the teacher.)

Equivalent Fractions

Main List

- + 1. Lights: Equivalent Fractions
- + 2. Pick a Tub
- + 3. Splash!
- + 4. Boxes: How Much Is Painted?
- + 5. Boxes: Equivalent Fractions Practice
- + 6. Sort Equivalent Fractions
- + 7. Boxes: Name Equivalent Fractions

Choose a lesson number. >

The "+" marks lessons you need to finish.

2 minutes left on this list. 18 minutes left in your session.

Fig. 1. Choice list of instructional lessons for a student at one point in an Equivalent Fractions module. The number of choices on a student's list can vary from one to many, depending on the design of the instructional module he is assigned to.

Equivalent Fractions

General List (Lessons from your Main List are included in case you want to do them.)

- + 1. Lights: Equivalent Fractions
- + 2. Pick a Tub
- + 3. Splash!
- + 4. Boxes: How Much Is Painted?
- + 5. Boxes: Equivalent Fractions Practice
- + 6. Sort Equivalent Fractions
- + 7. Boxes: Name Equivalent Fractions
- 8. Make-a-Monster with Equivalent Fractions
- 9. Paintings Library
- 10. Shoveling and Spider Web
- 11. Darts

Choose a lesson number. >

The "+" marks lessons you need to finish.

6 minutes left in your session.

Fig. 2. Expanded choice list. This list includes the instructional lessons the student is working on plus several general experience lessons.

Data Feedback to Teachers

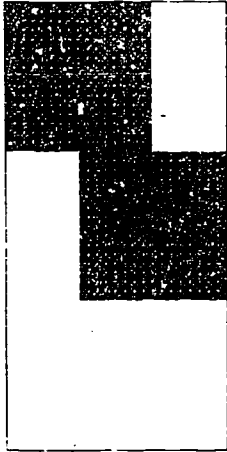
When the curriculum is used with the management system as intended, an on-line "grade book" which keeps track of student assignments, progress, and performance is available to the teacher at all times. The teacher can request a hard-copy printout of student assignment, progress, and performance data at any time. This request is made from any PLATO terminal. The printout is then automatically mailed to the teacher.

Supplementary Materials

A set of booklets and worksheets has been created to accompany the PLATO fractions curriculum. Each instructional module described in Appendix III has a corresponding set of materials. These materials are intended to help the teacher integrate the PLATO curriculum into the classroom mathematics program and help the student transfer the PLATO learning to pencil and paper. The worksheets are oriented toward work on individual skills, while each booklet provides a review and summing up of an entire module. When the booklets are used near the end of the appropriate module, the students are usually able to complete them with little difficulty and hence with the satisfaction of being able to do a task well. Some of the exercises include copies of PLATO displays with appropriate questions, so that the student is able to take home pictures that show what he is doing on PLATO. Figs. 3 and 4 show samples of the off-terminal materials.

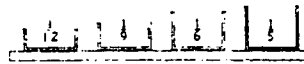
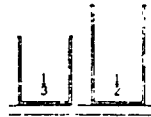
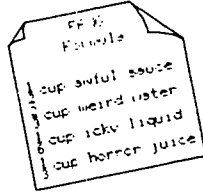
6

What fraction of this box is painted? _____



Draw lines on the box to show that your answer is right.

Choose one cup to do all your measuring.



Circle the cup you would choose. (Be sure it's one that can be used for all ingredients.)

How many times would you fill it to get $\frac{1}{2}$ cup of awful sauce? _____

How many times would you fill it for $\frac{3}{4}$ cup of weird water? _____

How many times for the icky liquid? _____

For the horror juice? _____

3

List the next 5 fractions in each equivalence set.

$\left\{ \frac{1}{2}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}} \right\}$

$\left\{ \frac{1}{4}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}} \right\}$

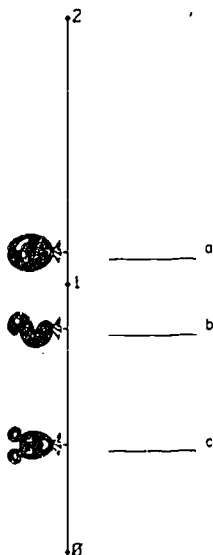
$\left\{ \frac{4}{5}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}} \right\}$

$\left\{ \frac{1}{7}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}} \right\}$

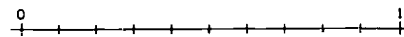
$\left\{ \frac{0}{1}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}} \right\}$

2

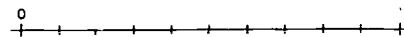
Where would you shoot the darts to hit the balloons?
Write a *DECIMAL* number beside each balloon.



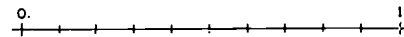
3



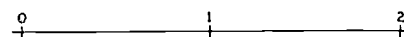
a. Show .6 on this number line.



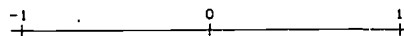
b. Show .65 on this number line.



c. Show .06 on this number line.



d. Show 1.4 on this number line.



e. Show -.8 on this number line.

TRY THIS ONE!

$$2 \times 3 \frac{1}{2} =$$



You could write $3 \frac{1}{2}$ two times and add them up.

Or you could think of 2 threes and 2 halves.



NOW TRY THESE!

Think of a way that is easy for you.
Write your answers in standard form.

$$5 \times 2 \frac{1}{2} =$$

$$4 \times 1 \frac{1}{2} =$$

$$3 \times \frac{1}{4} =$$

$$2 \times 5 \frac{1}{3} =$$

$$3 \times 2 \frac{1}{2} =$$

$$3 \times \frac{1}{3} =$$

$$5 \times \frac{1}{2} =$$

$$6 \times 1 \frac{1}{3} =$$

Fig. 4. Sample pages from some of the booklets that accompany the PLATO Fractions Curriculum. Each booklet provides a review and summary of a PLATO module.

History of Usage and Effectiveness

Beginning in 1973, the curriculum was developed in conjunction with use by students in public school classrooms. This has permitted the authors to tailor the curriculum to the needs of students and teachers in real educational settings. Since its first large-scale implementation during the 1974-75 school year, the curriculum has been a part of the daily classroom mathematics program for approximately 1100 intermediate grade students. It has also been used in various special education situations, among them hearing impairment, orthopedic impairment, and behavioral disorders.

In 1974-75, and again in 1975-76, investigators from Educational Testing Service in Princeton, New Jersey, conducted an external evaluation study of the PLATO Elementary Mathematics materials, in accordance with a contract from the National Science Foundation in Washington, D.C. Each study included about a dozen PLATO and a dozen non-PLATO classes. These classes, at the 4th, 5th, and 6th grade levels in eight different schools, included students from a wide range of scholastic abilities and socioeconomic levels.

A major component of each study was measurement of performance gains in each of the three PLATO Elementary Mathematics content "strands" -- Whole Numbers, Fractions, and Graphs -- by means of pre and post testing.

Data from these studies indicated that the Fractions strand was especially successful. Reporting to the American Educational Research Association on the results of the 1975-76 study, Educational Testing Service investigator Spencer Swinton (1978)¹ stated:

...used in a supplementary mode with students of appropriate readiness level, the PLATO elementary mathematics material yielded significant positive achievement and attitudinal effects. In the case of the fractions strand, these gains were large.

Average PLATO effects [on special, ETS- constructed, tests] were positive and highly significant for ...fractions at all three grades...

...analyses of both attitude and achievement instruments at the item level [supported] the conclusion that the fractions strand lessons had been highly effective.

It should be emphasized that these were classroom, not laboratory, studies. Teachers were in full control at all times, and they taught math in different ways and with different emphases. One result of this realistically decentralized approach was that the particular math topics covered varied widely among classes and students. In particular, computer-collected data show that the average PLATO student had computer assignments

¹Swinton, Spencer, "Outcomes of the PLATO Elementary Demonstration", paper presented at the Annual Meeting of the American Educational Research Association, 1978.

in fractions only about a third of the time and completed only about half of the available curriculum in fractions. Presumably, more nearly complete student coverage of the strand would have produced results even more favorable than those which Swinton found.

There is every reason to believe that most students would in fact have welcomed assignments to work further in the PLATO Fractions Curriculum. As Swinton indicates, results from attitude items specific to fractions were very positive in both years, and this is in agreement with both student and teacher reports.

In sum, the weight of the available evidence indicates that the PLATO sequence described in this manual is a powerful resource for the teacher facing the traditionally difficult problem of teaching fractions in the intermediate grades.

LESSON DESCRIPTIONS

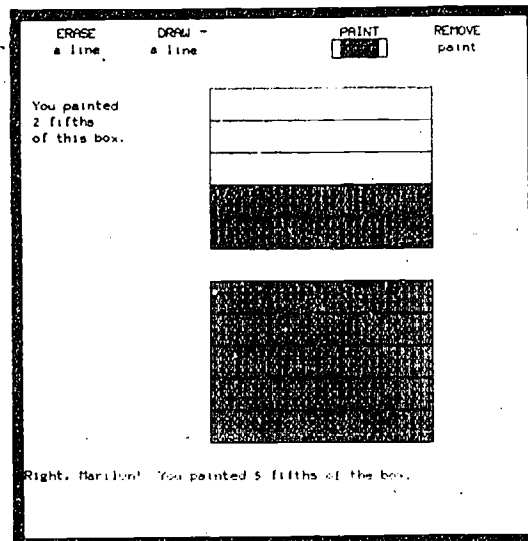
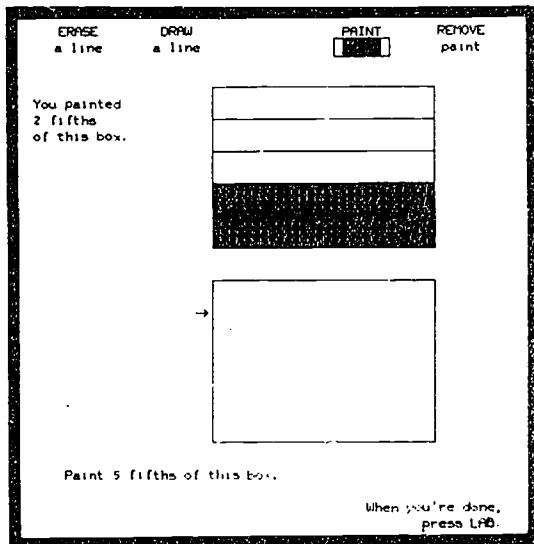
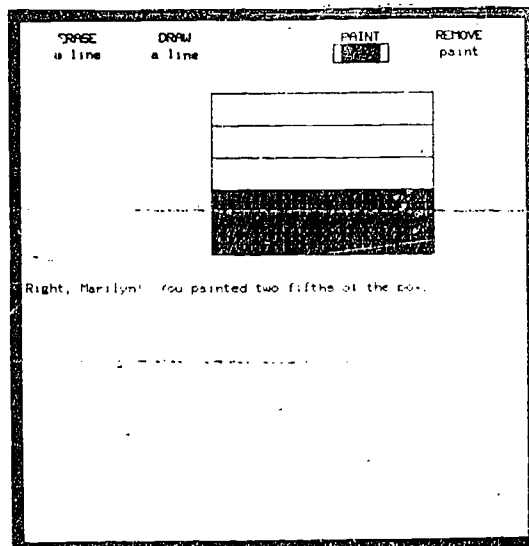
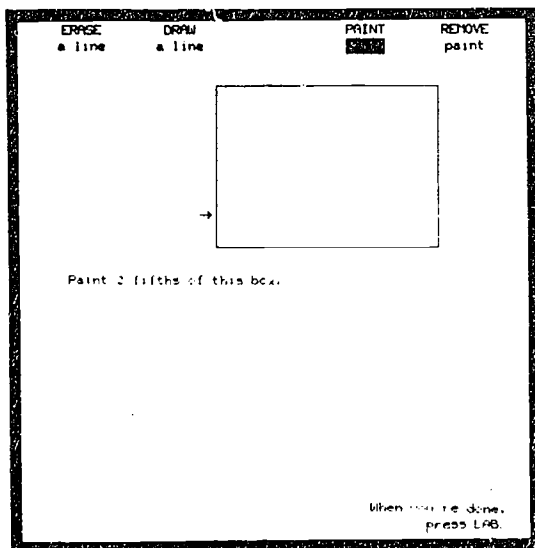
MEANING OF FRACTIONS

This set of lessons provides experience constructing, identifying, and comparing fractions, both in the discrete case and the continuous case. It is assumed that the concept of fractions has already been introduced.

Cut and Paint	A3
Fraction Notation	A4
Paint and Compare	A5
Paintings Library	A6
Lights	A7
Make-a-Monster	A8
Checkups	A9

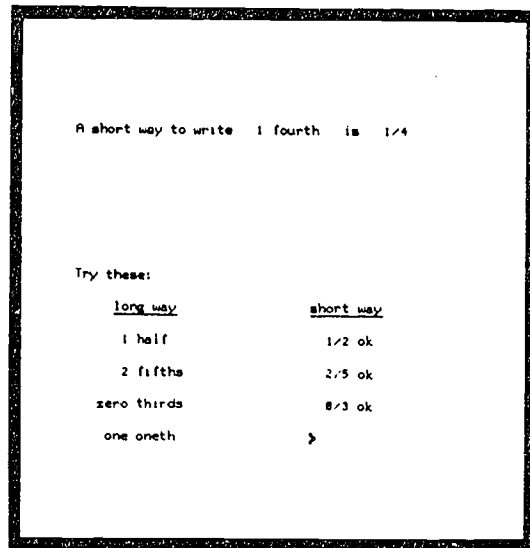
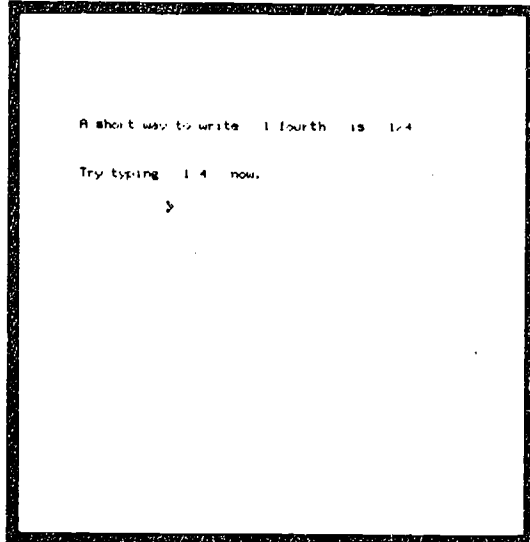
Note: Lessons that require the use of a touch panel are identified by the word "touch" after the title on the lesson description page.

Cut and Paint (touch)



- Purpose:
- 1) Let the student construct fractions of a region.
 - 2) Introduce numerators greater than 1.

Description: The student paints fractions of rectangular boxes. Fractions are written as "3 fourths," rather than " $3/4$." Difficulty adjusts to the student's performance.


Fraction Notation

Purpose: 1) Introduce $1/4$ as a short way to write 1 fourth.
 2) Show the student how to write $1/4$ on PLATO.

Description: PLATO explains that $1/4$ is a short way to write 1 fourth.
 The student practices writing fractions using the new notation.

Paint and Compare (touch)


ERASE a line DRAW a line PAINT ☐ REMOVE point



Touch the sentence you think is true, then press NEXT. (You can take a guess.)

$\frac{1}{2}$ of the box is more than $\frac{1}{3}$ of the box.
 $\frac{1}{3}$ of the box is more than $\frac{1}{2}$ of the box.
 $\frac{1}{2}$ of the box is just as much as $\frac{1}{3}$ of the box.

ERASE a line DRAW a line PAINT ☐ REMOVE point




Paint $\frac{1}{2}$ of the box above. When you're done, press LAB.

Let's find out....

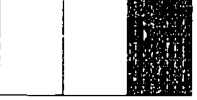
$\frac{1}{2}$ of the box is more than $\frac{1}{3}$ of the box.
 $\frac{1}{3}$ of the box is more than $\frac{1}{2}$ of the box.
 $\frac{1}{2}$ of the box is just as much as $\frac{1}{3}$ of the box.

ERASE a line DRAW a line PAINT ☐ REMOVE point

You painted $\frac{1}{2}$ of this box.



You painted $\frac{1}{3}$ of this box.




Do you want to change your guess? > yes


$\frac{1}{2}$ of the box is more than $\frac{1}{3}$ of the box.
 $\frac{1}{3}$ of the box is more than $\frac{1}{2}$ of the box.
 $\frac{1}{2}$ of the box is just as much as $\frac{1}{3}$ of the box.

ERASE a line DRAW a line PAINT ☐ REMOVE point

You painted $\frac{1}{2}$ of this box.



You painted $\frac{1}{3}$ of this box.



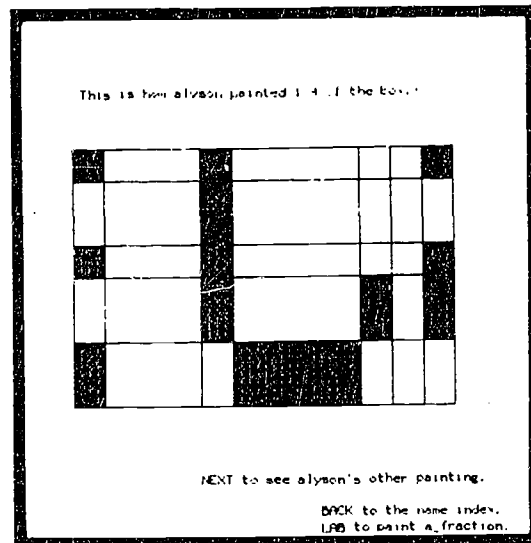
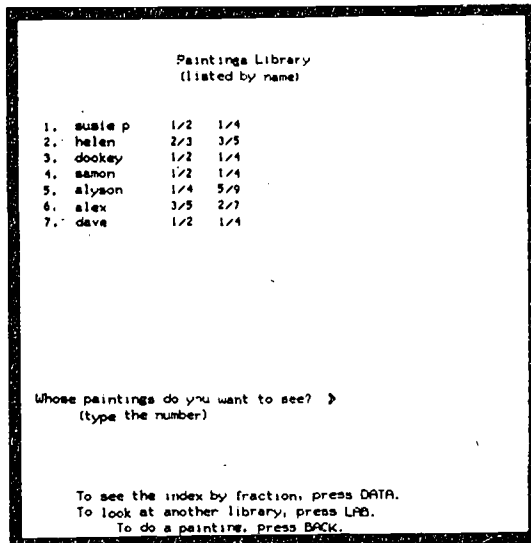
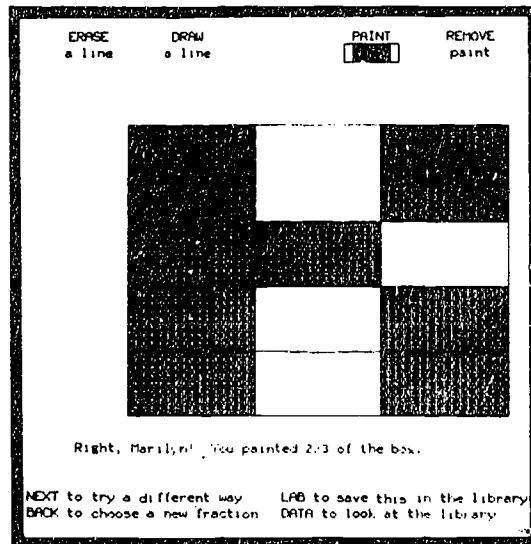
Plato agrees with you!

$\frac{1}{2}$ of the box is more than $\frac{1}{3}$ of the box.
 $\frac{1}{3}$ of the box is more than $\frac{1}{2}$ of the box.
 $\frac{1}{2}$ of the box is just as much as $\frac{1}{3}$ of the box.

Purpose: Introduce a method of comparing two fractions.

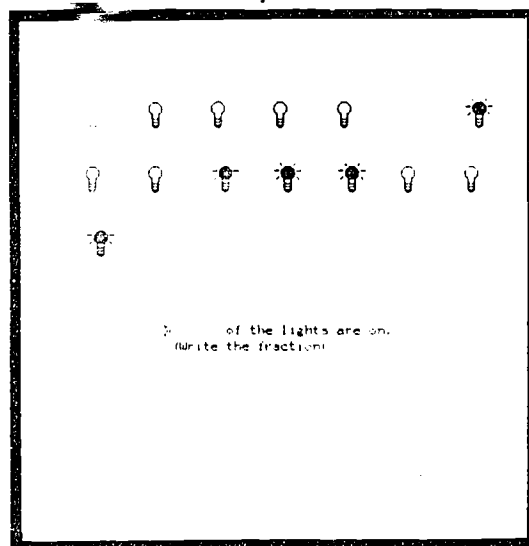
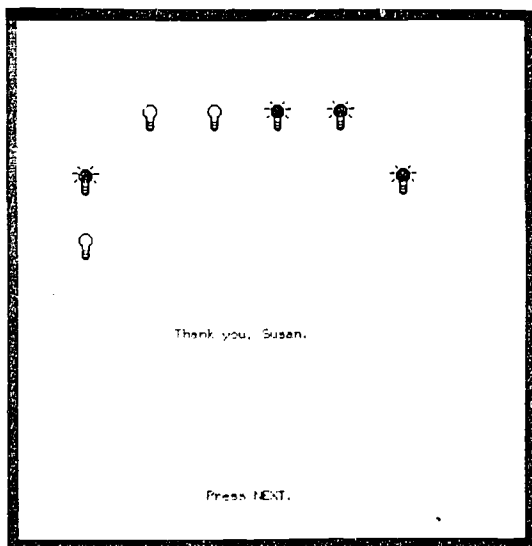
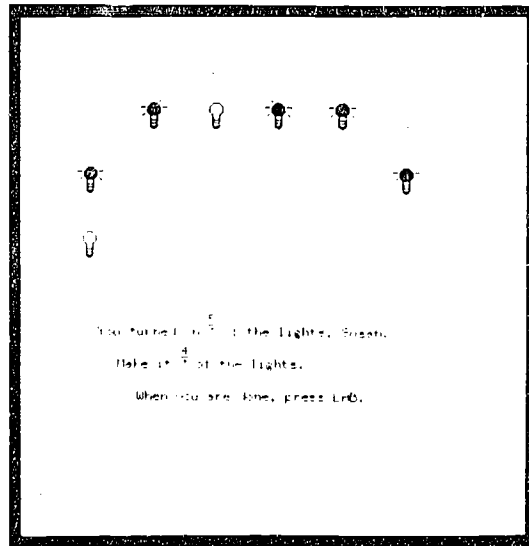
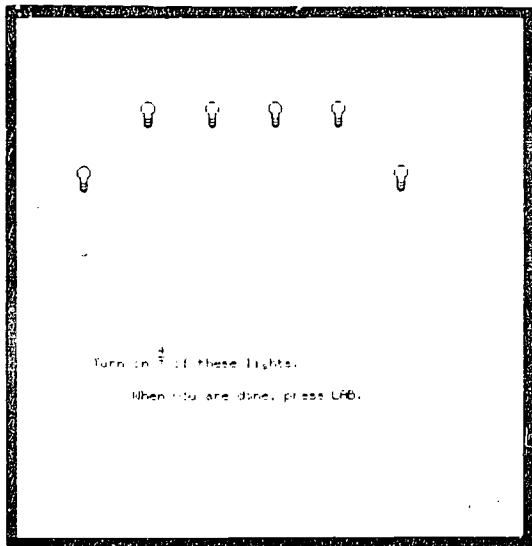
Description: The student predicts which of two fractions is bigger, then paints two identical boxes (one for each fraction) to check the prediction. The prediction can be changed after painting. Some of the problems involve equivalent fractions.

Paintings Library (touch)



- Purpose:**
- 1) Encourage painting a fraction of a box different ways.
 - 2) Initiate the idea of re-arranging the painted areas without changing the fraction painted.
 - 3) Provide opportunities to notice equivalent fractions.
 - 4) Let students share their work with others.

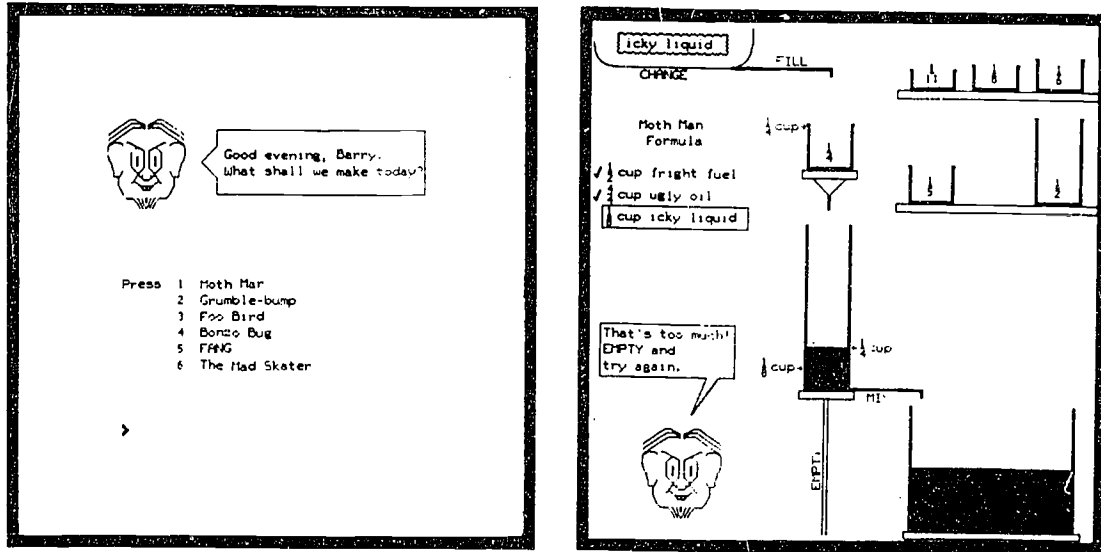
Description: The student can paint a box to keep in the library or look at paintings done by classmates. Due to space limitations, each student can keep only two paintings. A student can replace his or her paintings with new ones.

Lights (touch)

Purpose: Practice constructing and identifying fractions of a set of objects.

Description: The student turns on fractions of a set of light bulbs. After some success, PLATO turns on some lights and the student identifies the fraction represented. For this lesson the denominator of the fraction is always the number of lights in the set. The student is expected to move quickly to the next lesson ("Lights: Equivalent Fractions") where the denominator is different from the number of lights. Difficulty adjusts to the student's performance.

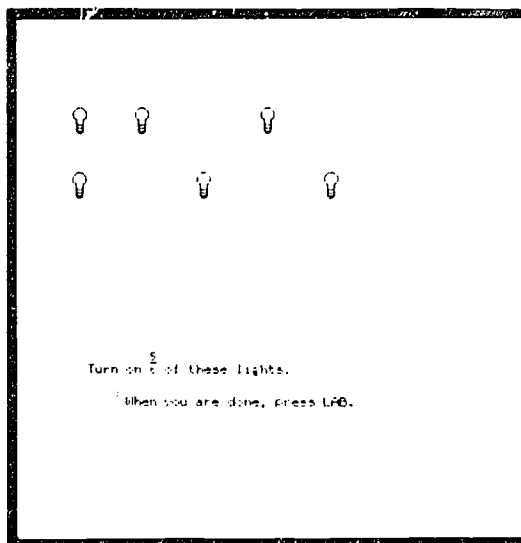
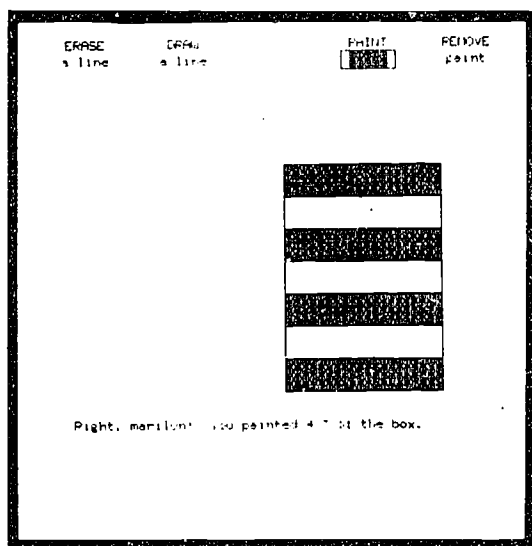
Make-a-Monster (touch)



- Purpose:
- 1) Re-emphasize that $\frac{3}{4}$ is 3 of a fourth.
 - 2) Provide experience with a new unit -- the cup.
 - 3) Familiarize the student with a model that will be useful later for equivalent fractions and common denominators.

Description: The student uses measuring cups to mix monster formulas in an eccentric scientist's laboratory. The available sizes of measuring cups have numerators of 1. To measure $\frac{3}{11}$, the student fills the $\frac{1}{11}$ cup 3 times.

Checkups



Short checkup sequences are available in the following lessons:

Cut and Paint
Lights

These sequences serve to review the previous work on the meaning of fractions and see if the student is ready for more advanced materials.

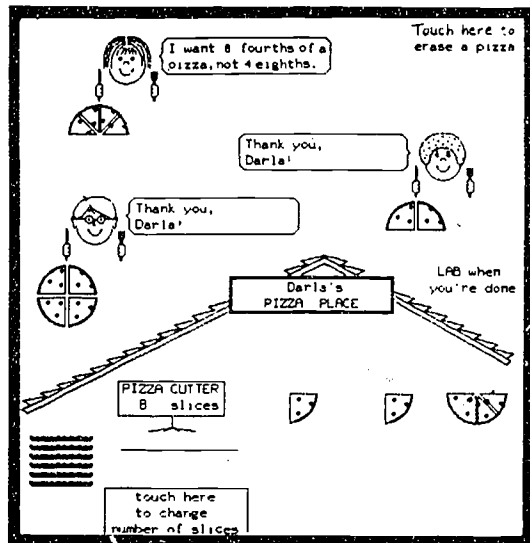
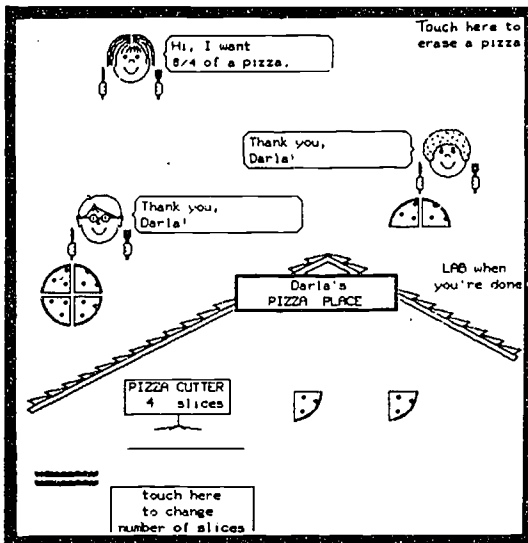
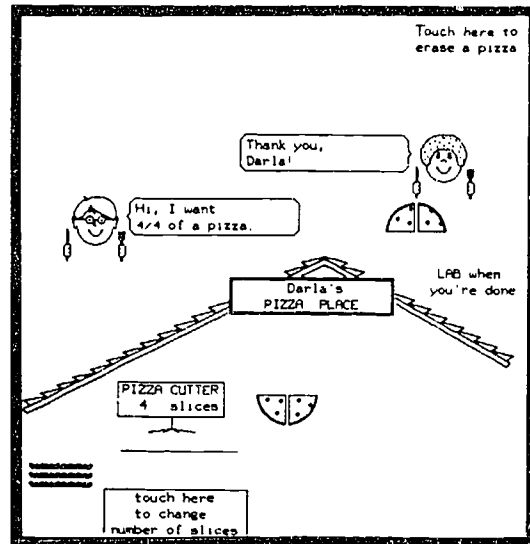
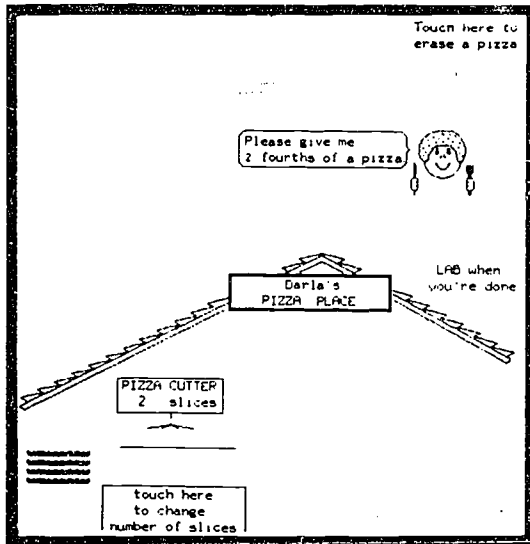
MIXED NUMBERS AND FRACTIONS GREATER THAN 1

Using a pizza model, this set of lessons introduces fractions greater than one and relates these fractions to mixed numbers. The terms "whole number," "fraction," and "mixed number" are presented. The student learns to convert mixed numbers to fractions and vice versa. Some work is done with renaming a mixed number as is done in addition and subtraction of mixed numbers. For example, 5 and $1/2$ is equivalent to 4 and $3/2$. One lesson illustrates the importance of the unit, and another introduces a new unit -- the turn.

Pizza: Fractions	B3
Pizza: Mixed Numbers Introduction	B4
Vocabulary: Whole Number, Fraction, Mixed Number	B5
Sort Numbers by Form	B6
Try These: Fractions and Mixed Numbers	B7
Try These: Equivalent Mixed Numbers	B8
Units	B9
Skywriting and Spider Web	B10
Checkups	B11

Note: Lessons that require the use of a touch panel are identified by the word "touch" after the title on the lesson description page.

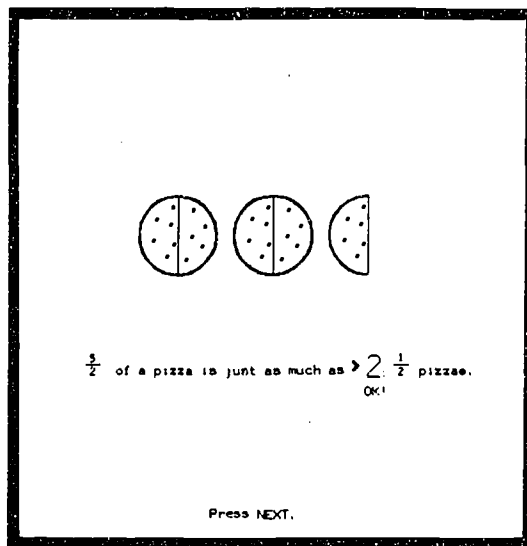
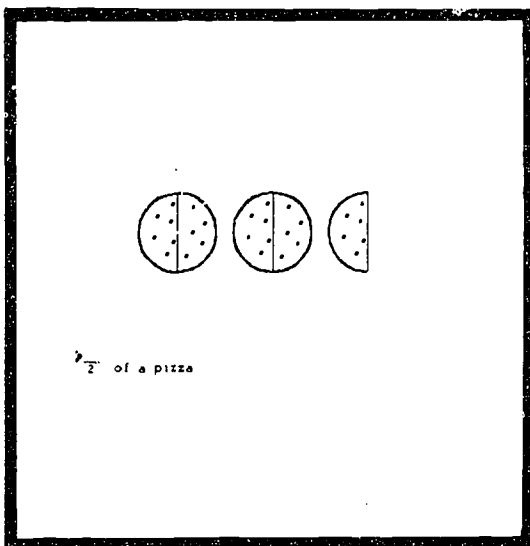
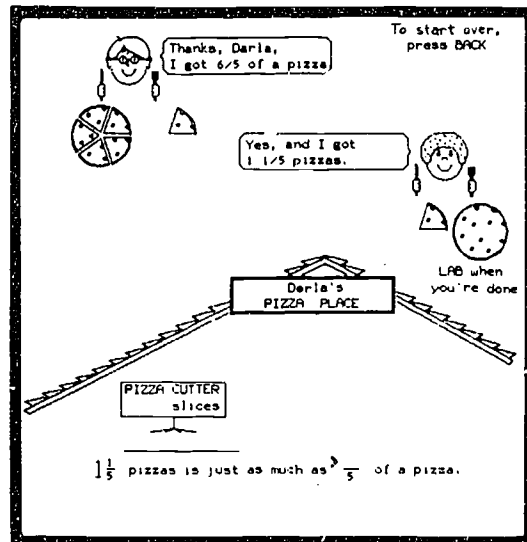
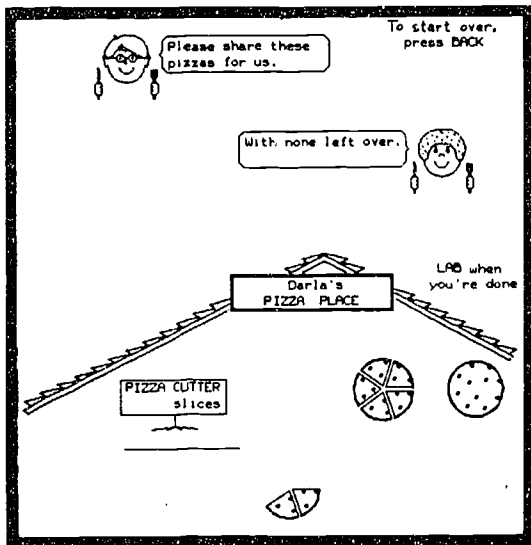
Pizza: Fractions (touch)



- Purpose: 1) Re-emphasize with physical model that $\frac{3}{4}$ is 3 of a fourth.
 2) Extend this to fractions greater than 1.

Description: The student operates a pizza place, serving kids who appear and ask for specific fractions of pizza. The problem sequence depends on the student's performance. It is designed to do fractions less than 1 until the student is familiar with the model, then lead into fractions equal to 1 and greater than 1.

Pizza: Mixed Numbers Introduction (touch)



Purpose: Introduce the idea that a fraction and a mixed number can name the same number, then provide practice naming a number with both a pure fraction and a mixed number.

Description: The student shares $1\frac{5}{3}$ pizzas (and similar amounts) fairly between two kids. The kids get $1\frac{1}{3}$ pizzas and $\frac{4}{3}$ of a pizza, respectively, so the conclusion is that $1\frac{1}{3}$ is equivalent to $\frac{4}{3}$. The problem sequence adjusts to the student's performance.

Later in the lesson the student fills in a fraction name and a mixed number name to describe a picture of pizzas. There is very specific step-through for errors. Difficulty adjusts to the student's performance. Higher level problems often have no picture as long as the student is doing well.

Vocabulary: Whole Number, Fraction, Mixed Number

These are whole numbers.

6 24 0

Name another whole number.
Marty: 5

These are fractions.

$\frac{25}{11}$ $\frac{3}{8}$ $\frac{4}{10}$
1/2 $\frac{8}{9}$

These are mixed numbers.

$3\frac{1}{2}$ $7\frac{3}{10}$ $1\frac{1}{3}$

$45\frac{8}{9}$ $0\frac{1}{10}$

These are whole numbers.

6 24 0

425 ok

These are fractions.

$\frac{25}{11}$ $\frac{3}{8}$ $\frac{4}{10}$
1/2 $\frac{8}{9}$
3/4 ok

These are mixed numbers.

$3\frac{1}{2}$ $7\frac{3}{10}$ $1\frac{1}{3}$

$45\frac{8}{9}$ $0\frac{1}{10}$

Name another mixed number, Marty. >

Why are these called mixed numbers?

I bet it's because they're all mixed up!

Maybe it's because they're made by mixing whole numbers and fractions together!

Purpose: Introduce vocabulary used in future lessons.

Description: The student is presented with examples of whole numbers, fractions, and mixed numbers, then asked to write one of each.

Sort Numbers by Form (touch)

WHOLE NUMBERS IN HERE	
FRACTIONS IN HERE	
MIXED NUMBERS IN HERE	
$1\frac{1}{2}$	19 $\frac{1}{4}$ 15 14
$\frac{2}{3}$	$8\frac{1}{4}$ 3 $4\frac{3}{8}$ 10

Touch here to
erase the ☐
around a number.

When you're done,
press -LFB-.

WHOLE NUMBERS IN HERE	
3 15 19 14	
FRACTIONS IN HERE	
$\frac{2}{3}$	$\frac{1}{4}$
MIXED NUMBERS IN HERE	
$1\frac{1}{2}$	$8\frac{1}{4}$ $4\frac{3}{8}$ 10

Touch here to
erase the ☐
around a number.

1 number is not in the right box.
When you're done,
press -LFB-.

WHOLE NUMBERS IN HERE	
3 15 19 14 10	
FRACTIONS IN HERE	
$\frac{2}{3}$	$\frac{1}{4}$
MIXED NUMBERS IN HERE	
$1\frac{1}{2}$	$8\frac{1}{4}$ $4\frac{3}{8}$

Touch here to
erase the ☐
around a number.

Very good!
Press -NEXT- for a new game.

Purpose: Practice distinguishing "fractions," "mixed numbers," and "whole numbers." This is necessary for understanding the vocabulary used in future lessons.

Description: Several numbers (whole numbers, fractions, and mixed numbers) are scattered on the screen with three boxes to sort them into: one each for whole numbers, fractions, and mixed numbers. The student sorts the numbers into the appropriate boxes.

Try These: Fractions and Mixed Numbers

TRY THESE!

$$\frac{9}{4} \Rightarrow \frac{1}{4} \quad \frac{1}{4} = 2\frac{1}{4}$$

$$\frac{3}{2} = 1\frac{8}{3} \quad \frac{1}{2} = 1\frac{1}{2}$$

TRY THESE!

$$\frac{9}{4} = 2\frac{1}{4} \text{ ok} \quad \frac{5}{4} = 2\frac{1}{4} \text{ no}$$

$$\frac{9}{4} = 2\frac{1}{4}$$

$$\frac{3}{3} = 1\frac{8}{3} \text{ ok} \quad \frac{1}{2} = 1\frac{1}{2}$$

Purpose: Provide practice converting fractions to mixed numbers and vice versa. This is for students who already have a basic understanding, but need practice.

Description: There are four problems per page, and no pictures or models. Problems are like $2\frac{1}{3} = \gg /3$. If the student is wrong twice on a problem, PLATO gives the answer to it. Difficulty adjusts to the student's performance.

Try These: Equivalent Mixed Numbers

TRY THESE!

$$5\frac{2}{3} = 4\frac{4}{3} \quad 1\frac{1}{3} = 2\frac{1}{3}$$

$$2\frac{4}{3} = \frac{2}{3} \quad 4\frac{3}{4} = 3\frac{6}{4}$$

TRY THESE!

$$5\frac{2}{3} = 4\frac{4}{3} \text{ ok} \quad 1\frac{1}{3} = 2\frac{1}{3} \text{ no}$$

$$1\frac{4}{3} = 2\frac{1}{3}$$

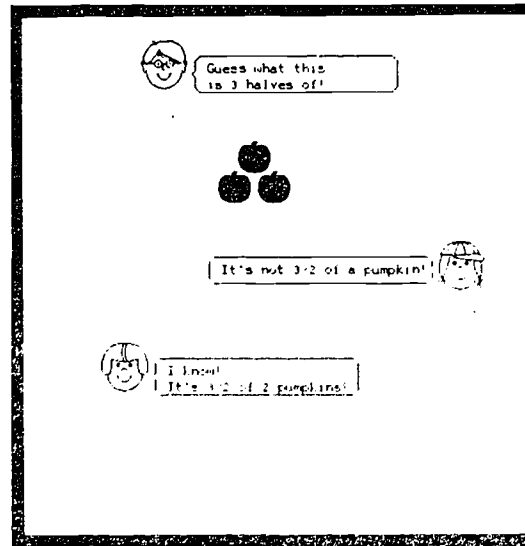
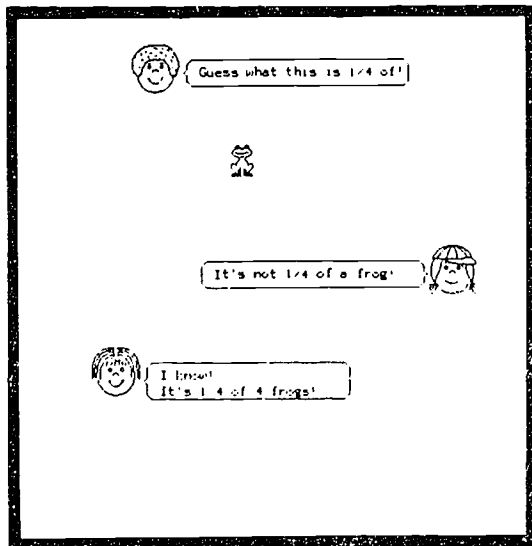
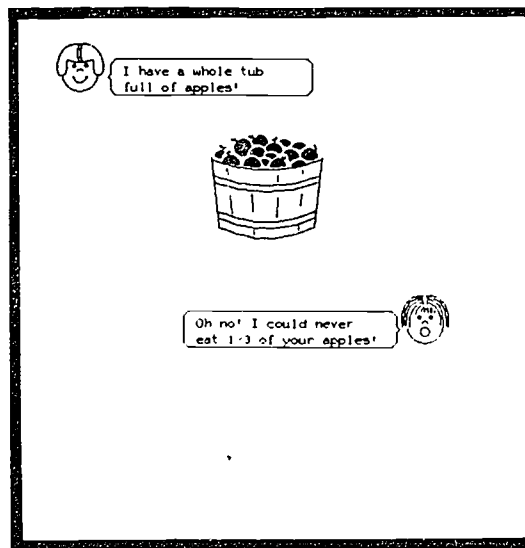
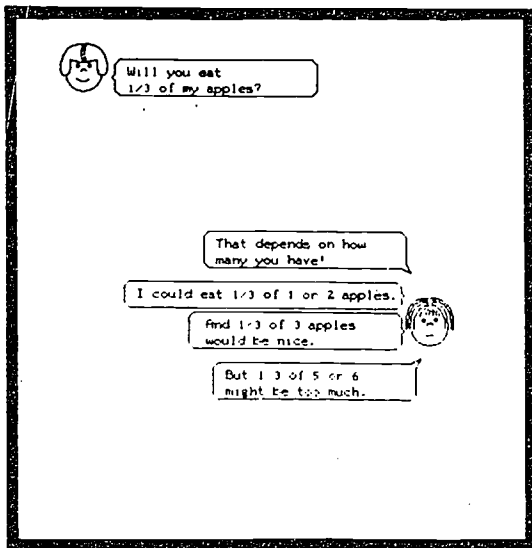
$$2\frac{8}{3} = 1\frac{2}{3} \text{ ok} \quad 4\frac{3}{4} = 3\frac{6}{4} \text{ ok}$$

Press NEXT.

Purpose; Provide practice renaming mixed numbers as done in addition and subtraction of mixed numbers. This is for students who have a basic understanding, but need practice.

Description: There are four problems per page, and no pictures or models. Problems are like $4\frac{1}{2} = 3\frac{2}{2}$. If the student is wrong twice on a problem, PLATO gives the answer to it. Difficulty adjusts to the student's performance.

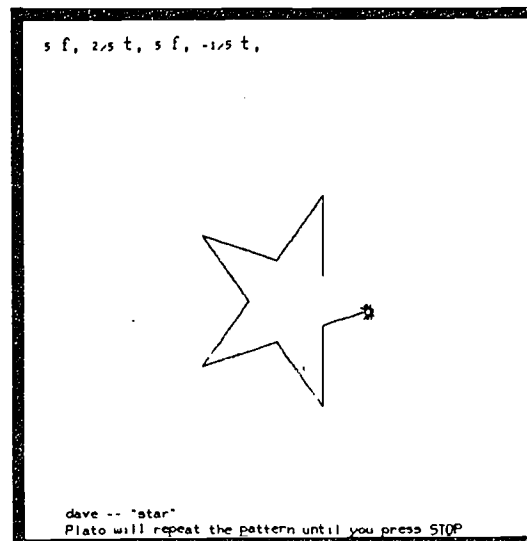
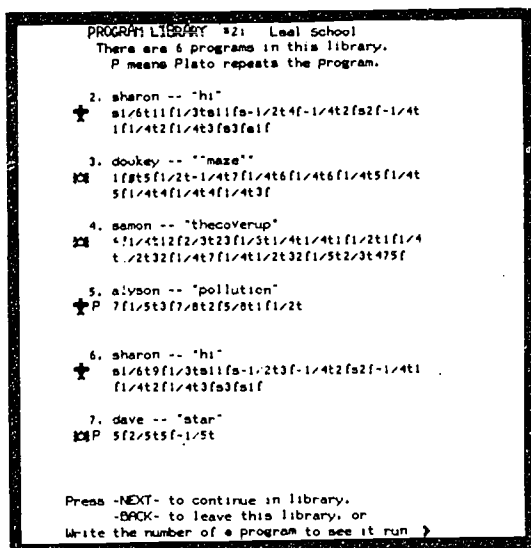
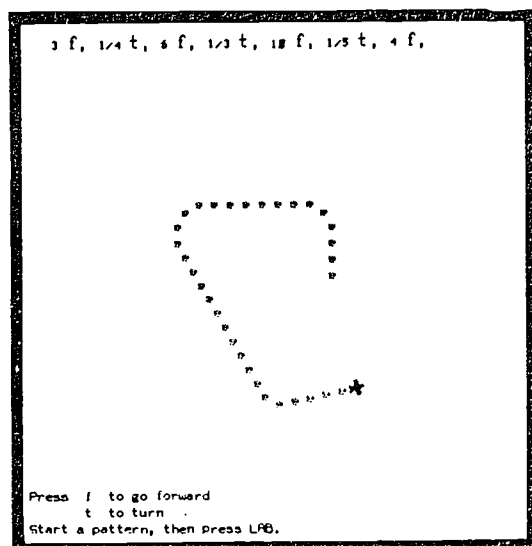
Units



Purpose: Point out the importance of the unit. For example, $\frac{1}{2}$ of one unit can be the same as $\frac{3}{4}$ of another unit.

Description: There are ten independent pages in this lesson. On each page some kids on the screen discuss a situation involving units. The student fills in appropriate parts of the discussion.

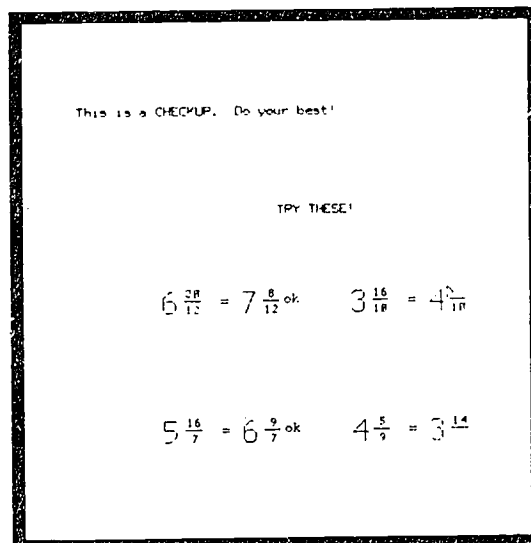
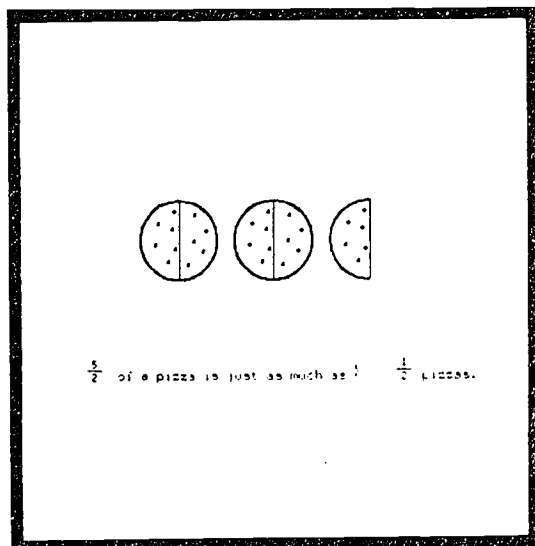
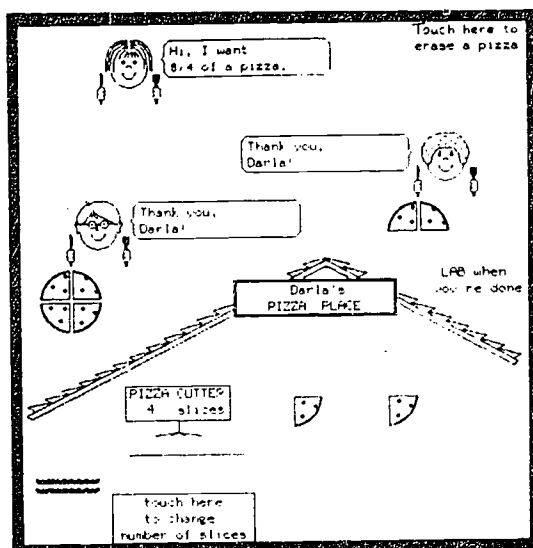
Skywriting and Spider Web



- Purpose:
- 1) Provide experience with a new unit -- a turn.
 - 2) Provide experience with very basic programming ideas.
 - 3) Let students share their work with other students through the program library.

Description: The student writes a program to move an airplane (or spider) around the screen, leaving a trail of smoke (or web). A program consists of a sequence of "forwards" and "turns." Each student can save one program in the library and replace it at any time with a new one.

Checkups



Short checkup sequences are available in the following lessons:

Pizza: Fractions

Pizza: Mixed Numbers Introduction

Try These: Equivalent Mixed Numbers

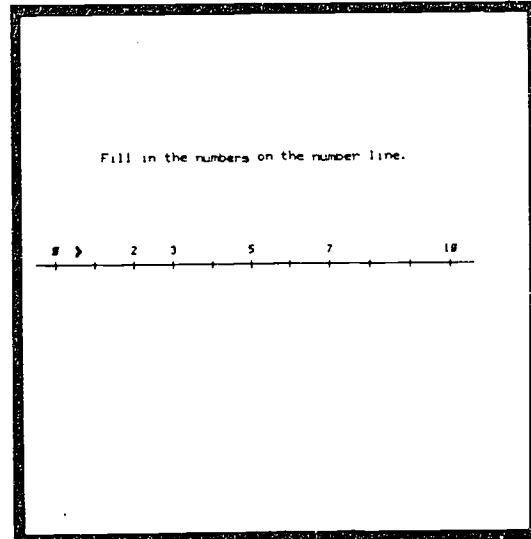
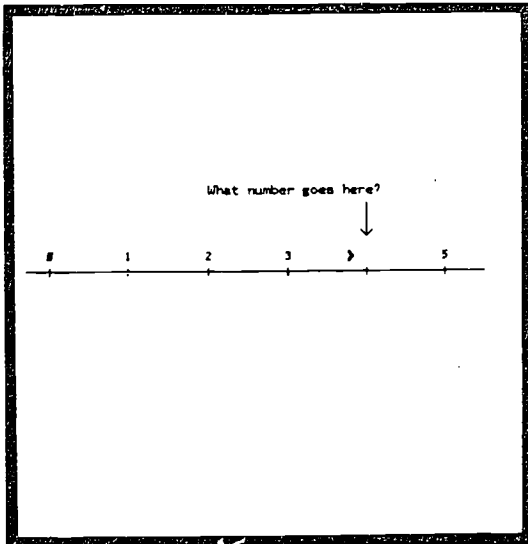
These sequences serve to review the previous work on mixed numbers and fractions greater than 1 and see if the student is ready for more advanced material.

FRACTIONS AND MIXED NUMBERS ON THE NUMBER LINE

This set of lessons introduces the number line and provides experience with placement of fractions and mixed numbers on the number line. Experience with fractional distances and related ideas is also provided in various game formats.

Number Line Introduction	C3
Mixed Number Notation	C4
Darts	C5
Pick a Tub	C6
Splash!	C7
Torpedo	C8
Checkup: Darts	C9

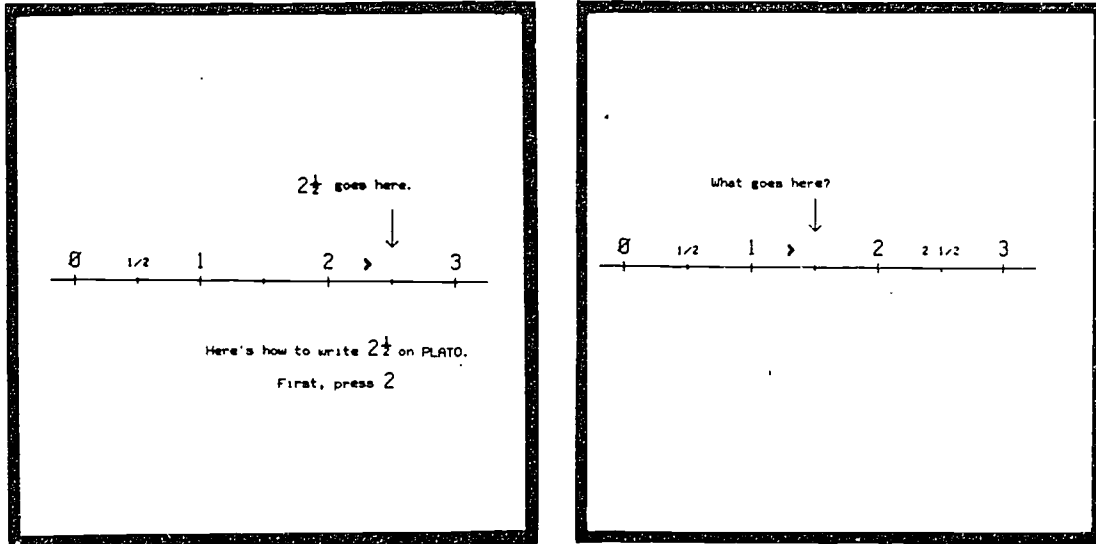
Note: Lessons that require the use of a touch panel are identified by the word "touch" after the title on the lesson description page.

Number Line Introduction

Purpose: Familiarize the student with whole numbers on the number line.

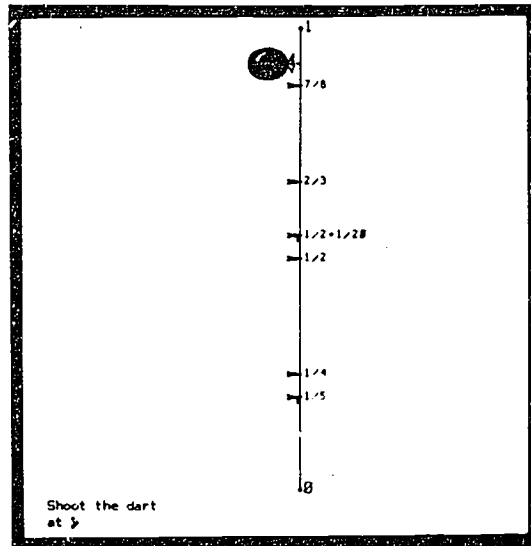
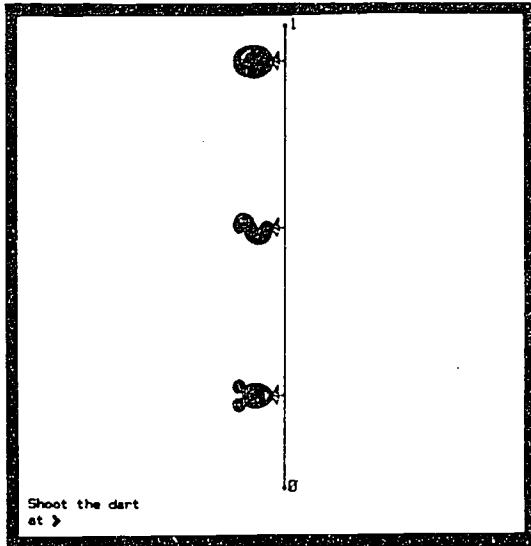
Description: The student fills in whole numbers on a number line. Difficulty adjusts to the student's performance.

Mixed Number Notation



Purpose: Show the student how to write mixed numbers on PLATO.

Description: PLATO shows how to write $2 \frac{1}{2}$. The student writes $\frac{1}{2}$, $1 \frac{1}{2}$, and $2 \frac{1}{2}$ to mark points on a number line that already has the whole numbers labelled.

Darts

Purpose: Experience locating fractions and mixed numbers on the number line.

Description: The student writes numbers or expressions to shoot darts at balloons tied to a number line. Difficulty adjusts to the student's performance. Higher levels have fewer integers on the line and/or smaller balloons to shoot at.

Pick a Tub (touch)

Helen's turn!

$\frac{3}{2}$	$\frac{17}{3}$	$\frac{11}{2}$	$\frac{18}{3}$	$\frac{17}{3}$
$\frac{14}{3}$	$\frac{5}{2}$	$\frac{11}{3}$	$\frac{3}{2}$	$\frac{11}{2}$

Touch a fraction and a tub, then press NEXT.

H for Helen
a for Andy

For help, press HELP.

Helen's turn!

$\frac{3}{2}$	$\frac{17}{3}$	$\frac{11}{2}$	$\frac{18}{3}$	$\frac{17}{3}$
$\frac{14}{3}$	$\frac{5}{2}$	$\frac{11}{3}$	$\frac{3}{2}$	$\frac{11}{2}$

Press NEXT for a new ball.

H for Helen
a for Andy

For help, press HELP.

Andy's turn!

$\frac{11}{2}$	$\frac{18}{3}$
$\frac{11}{2}$	

Touch a fraction and a tub, then press NEXT.

H for Helen
a for Andy

For help, press HELP.

Andy's turn!

$\frac{11}{2}$	$\frac{18}{3}$
$\frac{11}{2}$	

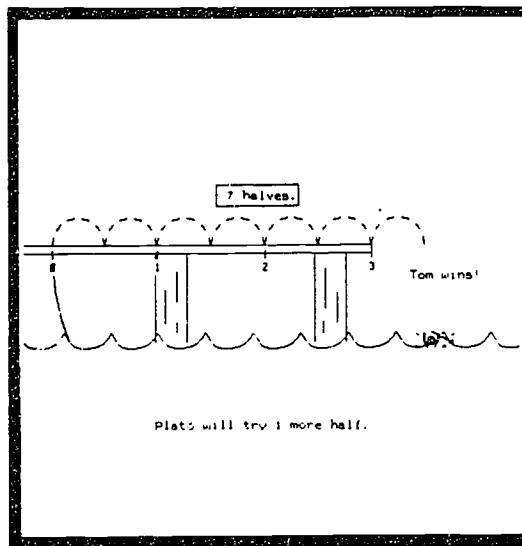
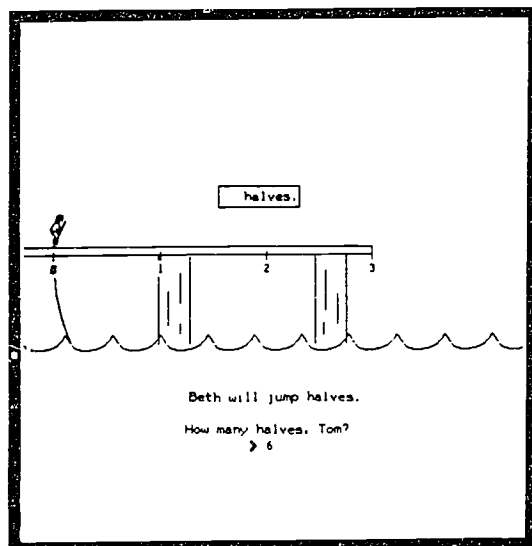
Press NEXT for a new ball.

H for Helen
a for Andy

For help, press HELP.

Purpose: Provide experience estimating positions of pure fractions on a number line.

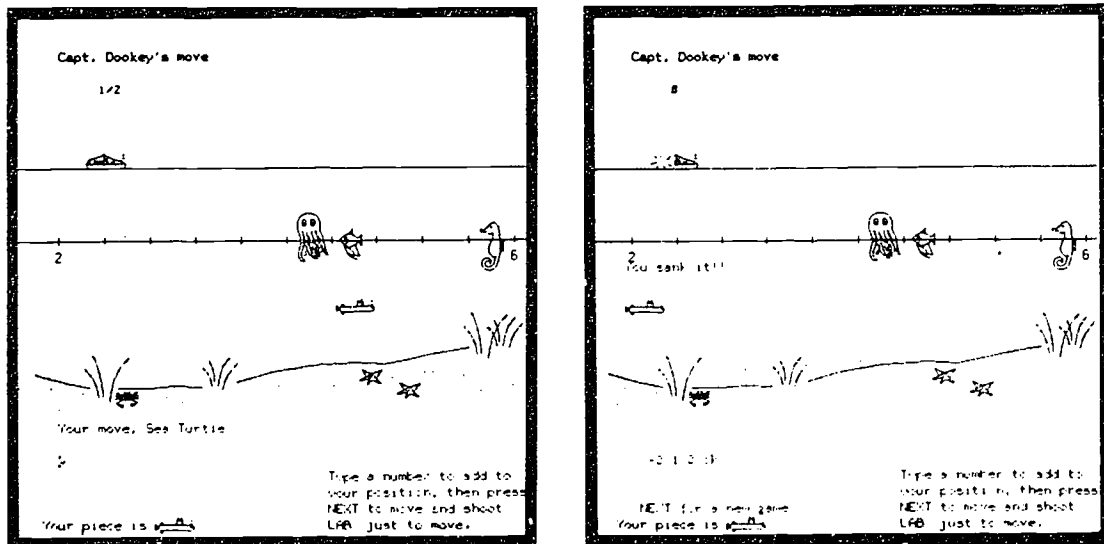
Description: This is a game for one or two players. A ball is on a number line with tubs underneath, and ten fractions are given. Each turn consists of touching a fraction and a tub. The ball bounces the fraction distance and falls into a tub. If the student chose the right tub, the ball is marked with his or her initial. Difficulty adjusts to performance.

Splash!

Purpose: Provide general experience relating and converting fractions to integers and mixed numbers.

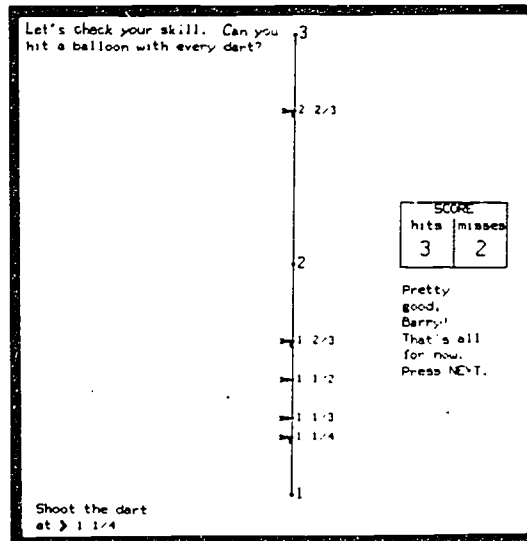
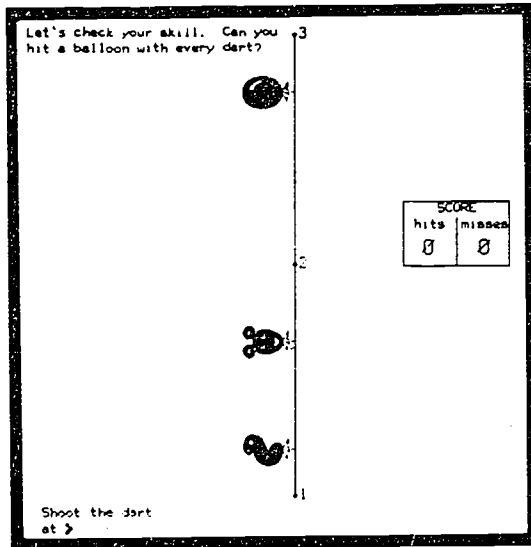
Description: This is a game for one or two players. A kid on a pogo stick tries to hop to the end of a dock without falling into the water. The student tells the kid how far to go. Difficulty adjusts to the student's performance.

Torpedo



Purpose: Practice estimating fractional distances on a number line.

Description: This is an interterminal number line game. Two students compete. One controls a boat, the other a submarine. The object is to move to a point above (or below) the opponent's piece and shoot it. The number line varies from game to game.

Checkup: Darts

Purpose: Review fractions on the number line and see if the student is ready for further number line work.

Description: The student plays two games of "Darts." Each game has three balloons to pop, all of which can be hit with denominators less than 7. PLATO displays the student's score of "hits" and "misses."

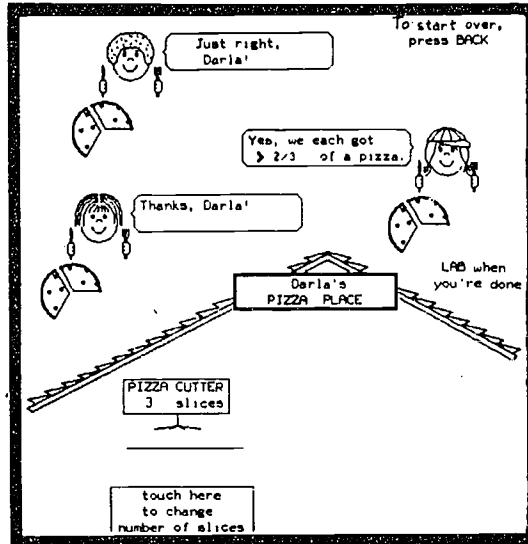
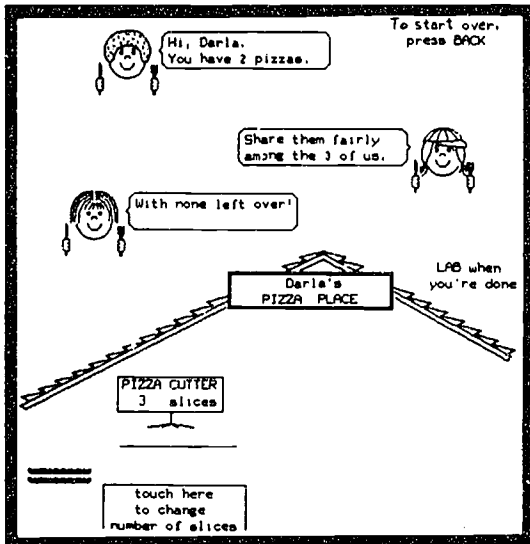
RELATING FRACTIONS TO DIVISION

This lesson relates fractions to division by showing that x pizzas divided by y kids gives x/y pizzas per kid. The lesson uses a "box" and a "hex" as variables to generalize the rule $x \div y = x/y$.

Pizza: 2 Pizzas for 3 Kids D3

Note: Lessons that require the use of a touch panel are identified by the word "touch" after the title on the lesson description page.

2 Pizzas for 3 Kids (touch)



2 pizzas for 5 kids. Each kid gets $\frac{2}{5}$ of a pizza.

7 pizzas - 4 kids = $\frac{7}{4}$ of a pizza for each kid.

17 pizzas - 9 kids = $\frac{17}{9}$ of a pizza for each kid.

7 pizzas - 5 kids = $\frac{7}{5}$ of a pizza for each kid.

pizzas \div kids = $\frac{\quad}{\quad}$ of a pizza for each kid.

Purpose: Introduce the idea that $x \div y$ is equivalent to the fraction x/y .

Description: The student has two pizzas which must be divided equally among 3 kids. This shows that 2 pizzas divided by 3 kids is $\frac{2}{3}$ pizzas per kid. After several similar problems, the student fills in "short stories" of the format: "2 pizzas for 3 kids. Each one gets $\frac{2}{3}$ of a pizza." The student then uses and to generalize the rule $\square \div \square = \square / \square$.

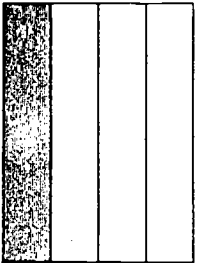
EQUIVALENT FRACTIONS INSTRUCTION WITH A MODEL

This set of lessons develops a method for constructing fractions equivalent to a given fraction. The basic model is a rectangular box, some fraction of which has been painted. The student learns to find equivalent fractions by dividing the box into smaller equal pieces. For example, if the box is cut into fourths, and one fourth is painted, then the student can divide each fourth to show that $\frac{2}{8}$ of the box is painted. This method is used to show that multiplying the numerator and denominator of a fraction by the same number creates another name for the fraction. This multiplication method is used to list equivalence sets for fractions. The term "equivalence set" for a fraction is used to mean the ordered list produced by multiplying numerator and denominator by the set of integers, beginning with 1.

Boxes: Equivalent Fractions	E3
Boxes: Equivalent Fractions Practice	E4
Boxes: Name Equivalent Fractions	E5
Equivalent Fractions Generalized	E6
Checkups	E7

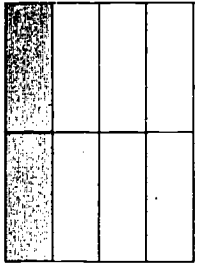
Note: Lessons that require the use of a touch panel are identified by the word "touch" after the title on the lesson description page.

Boxes: Equivalent Fractions



Think of cutting the box into 8 equal pieces.
How many of the pieces would be painted? >

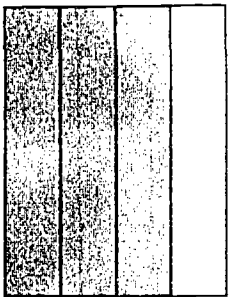
$\frac{1}{4}$ of the box is painted.



Think of cutting the box into 8 equal pieces.
How many of the pieces would be painted? 2

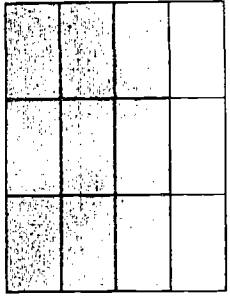
$\frac{1}{4}$ of the box is painted.
 $\frac{2}{8}$ of the box is painted.

So $\frac{2}{8}$ of the box is painted.



> $\frac{9}{12}$ of the box is painted.

$\frac{3}{4}$ of the box is painted.
 $\frac{6}{8}$ of the box is painted.



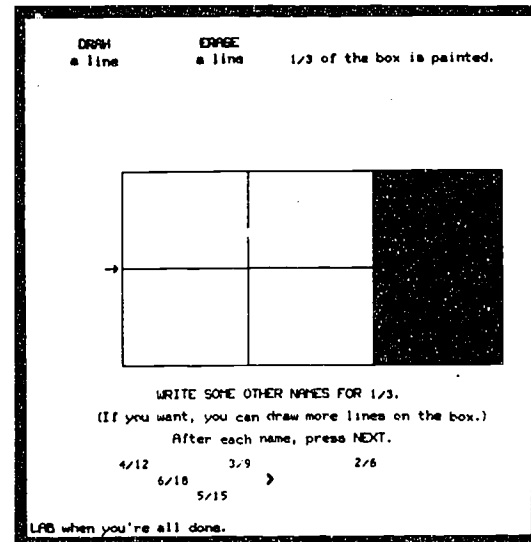
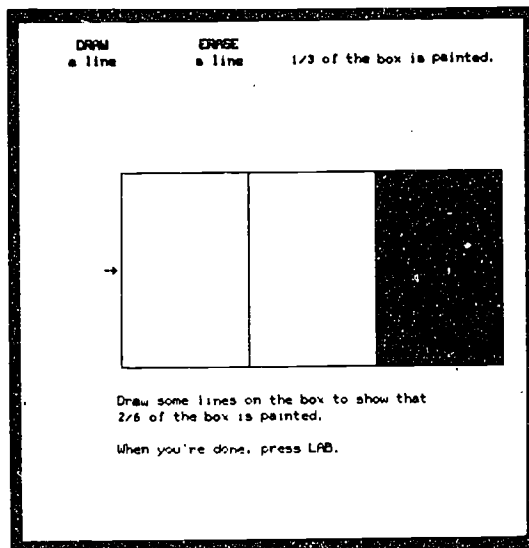
$\frac{9}{12}$ of the box is painted.

$\frac{3}{4}$ of the box is painted.
 $\frac{6}{8}$ of the box is painted.
 $\frac{9}{12}$ of the box is painted.

Purpose: Introduce a method for constructing fractions equivalent to a given fraction.

Description: PLATO paints a fraction of a box. The student is stepped through the process of cutting the box into more pieces to find another name for the fraction. Difficulty adjusts to the student's performance. The first problem gives a very specific step-through; later ones move faster and give less step-through.

Boxes: Equivalent Fractions Practice (touch)



Purpose: Practice naming equivalent fractions with a familiar model available for help.

Description: PLATO paints a fraction of a box, and the student writes other names for that fraction. (The student can draw and erase lines on the box.) Some problems ask the student to draw lines on the box to show that two fractions are equivalent. Difficulty adjusts to the student's performance.

Boxes: Name Equivalent Fractions (touch)

Names for $\frac{4}{5}$ fifths:

$\left\{ \frac{4}{5}, \right\}$ $\frac{4}{5}$

--	--	--	--	--

What is another name
for $\frac{4}{5}$? >

Touch here to
change your mind.

Names for $\frac{4}{5}$ fifths:

$\left\{ \frac{4}{5}, \frac{8}{10}, \frac{3}{4}, \frac{48}{60}, \frac{16}{20}, \frac{12}{15} \right\}$ $\frac{4}{5}$

--	--	--	--	--

Are all the fractions
names for $\frac{4}{5}$? >

$\frac{8}{10}$

--	--	--	--	--

$\frac{3}{4}$

--	--	--	--	--

$\frac{48}{60}$

$\frac{16}{20}$

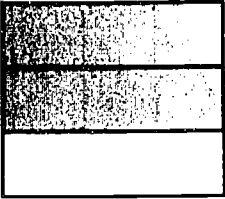
$\frac{12}{15}$

Purpose: Practice naming equivalent fractions with visual feedback comparing all of the named fractions.

Description: PLATO paints a fraction of a box. The student names fractions equivalent to the given one. Each named fraction is painted on a new box so that all of the fractions can be compared. The student removes any fractions that he or she decides are not equivalent to the given one. Difficulty adjusts to the student's performance.

Equivalent Fractions Generalized

$\frac{2}{3}$ of the box is painted.




$$\frac{2 \div 2}{3 \div 2} = \frac{4}{6}$$


$$\frac{2 \div 5}{3 \div 5} = \frac{10}{15}$$

$$\frac{2 \div 4}{3 \div 4} = \frac{8}{12}$$


$\frac{4}{6}$ is a name for $\frac{2}{3}$.



I don't think so. Show me why.



$\frac{2}{3}$ of the box is painted.



$$\frac{2 \div 2}{3 \div 2} = \frac{4}{6}$$

$$\frac{2 \div 5}{3 \div 5} = \frac{10}{15}$$

$$\frac{2 \div 4}{3 \div 4} = \frac{8}{12}$$

$\frac{8}{12}$ is a name for $\frac{2}{3}$.

PLATO just cut each piece into 4 pieces.

Now there are 4 times as many pieces.


And 4 times as many painted pieces.

Now there are 2×4 painted pieces.

$$\frac{2 \times 4}{3 \times 4} = \frac{8}{12} \text{ of the box is painted.}$$

Now there are 3×4 pieces.

I see! $\frac{8}{12}$ really is a name for $\frac{2}{3}$.




Let's list the names in order.

Names for $\frac{1}{2}$

$\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \dots$

$\frac{1 \div 6}{2 \div 6} = \frac{6}{12}$



List the equivalence set for $\frac{1}{2}$.

Write the next fraction.

$\left\{ \frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \dots \right\}$

- Purpose:**
1. Present a systematic method for finding fractions equivalent to a given fraction.
 2. Practice naming an equivalence set in order, starting with simplest terms. This skill will be useful later for systematically finding a common denominator for addition and subtraction of fractions.

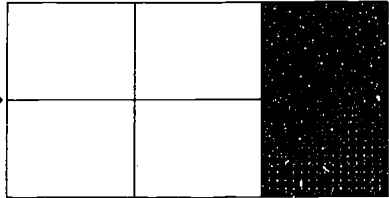
Description: The first part of the lesson demonstrates that other names for a fraction can be found by multiplying the numerator and the denominator both by the same number. Kids on the screen question the method. PLATO justifies it for various cases by making more lines on a painted box and giving a short explanation. In the second part of the lesson, the student uses this method to list equivalence sets for given fractions. Difficulty adjusts to the student's performance. Lower difficulty levels offer much help. Higher levels encourage the student to work without PLATO's help.

Checkups

DRAW
a line

ERASE
a line

$\frac{1}{3}$ of the box is painted.



WRITE SOME OTHER NAMES FOR $\frac{1}{3}$.
If you want, you can draw more lines on the box.
After each name, press NEXT.

List the equivalence set for $\frac{1}{3}$.
Write the next fraction.

($\frac{2}{6}$, $\frac{4}{12}$, $\frac{8}{24}$)

Short checkup sequences are available in the following lessons:

Boxes: Equivalent Fractions Practice
Equivalent Fractions Generalized

These sequences serve to review the previous work on equivalent fractions and see if the student is ready for more advanced material.

REDUCING FRACTIONS TO SIMPLEST TERMS

This set of lessons defines "simplest terms" and develops a systematic method for reducing fractions to simplest terms. The simplest terms name for a fraction is defined to be the name that has the smallest numerator and denominator. It is assumed that the student is familiar with equivalence sets for fractions as presented in the "Equivalent Fractions Instruction with a Model" lessons. The student learns to find the simplest terms name by dividing out common factors of the numerator and denominator. There is some discussion about why this method works. The student is encouraged to experiment with factors and divisibility relationships of equivalent fractions.

Simplest Terms	F3
Equivalence Experience	F4
Checkup: Simplest Terms	F5

Note: Lessons that require the use of a touch panel are identified by the word "touch" after the title on the lesson description page.

Simplest Terms

These fractions are all names for the same number.

$\left\{ \frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{6}{12}, \frac{7}{14}, \frac{8}{16}, \dots \right\}$

$\frac{1}{2}$ is the simplest terms name.

These fractions are all names for the same number.

$\left\{ \frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \frac{5}{15}, \frac{6}{18}, \frac{7}{21}, \dots \right\}$

$\frac{1}{3}$ is the simplest terms name.

These fractions are all names for the same number.

$\left\{ \frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \frac{5}{20}, \frac{6}{24}, \frac{7}{28}, \dots \right\}$

What is the simplest terms name? $\frac{1}{4}$ ok

Right! $\frac{1}{4}$ is the simplest terms name for $\frac{1}{4}$.

And for $\frac{3}{12}$, too!

The simplest terms name is the name with the smallest numerator and denominator.

Here are some equivalence sets.

$\left\{ \frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{6}{12}, \frac{7}{14}, \frac{8}{16}, \dots \right\}$

$\left\{ \frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \frac{5}{15}, \frac{6}{18}, \frac{7}{21}, \dots \right\}$

$\left\{ \frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \frac{5}{20}, \frac{6}{24}, \frac{7}{28}, \dots \right\}$

What is the simplest terms name for $\frac{1}{2}$? $\frac{1}{2}$ ok

What is the simplest terms name for $\frac{1}{3}$? $\frac{1}{3}$ ok

What is the simplest terms name for $\frac{1}{4}$? $\frac{1}{4}$ ok

What is the simplest terms name for $\frac{1}{5}$? $\frac{1}{5}$ ok

You don't really need the equivalence set!

We'll help you find the simplest terms name for $\frac{6}{12}$ without the equivalence set.

First we'll fill these boxes with a number that makes each division come out even.

$\frac{6}{12} = \frac{\boxed{2}}{\boxed{2}}$

Let's use 2. It works.

Find the simplest terms name for $\frac{16}{24}$.

Fill both boxes with a number that makes each division come out even.

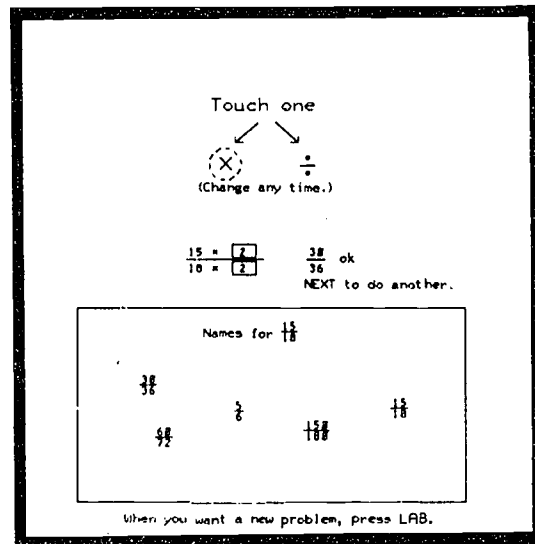
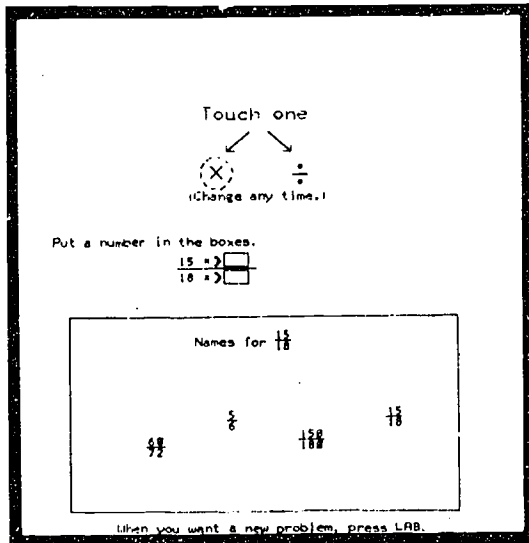
$\frac{16}{24} = \frac{\boxed{4}}{\boxed{6}}$

Purpose: Give the student a somewhat rough definition of the "simplest terms" name for a fraction, then present and practice a systematic method for finding the simplest terms name for any fraction.

Description: Given an ordered equivalence set for a fraction, PLATO points out the simplest terms name. After a few such examples, PLATO presents an equivalence set and asks the student to identify the simplest terms name. Kids on the screen point out that the simplest terms name is the name with the smallest numerator and denominator. Practice examples follow.

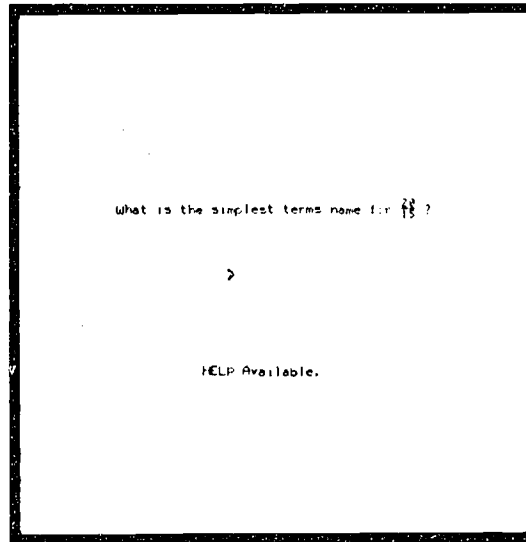
Kids on the screen then suggest that the equivalence set is not necessary for finding the simplest terms name for a fraction. For given fractions the kids help the student divide out common factors of the numerator and denominator to find the simplest terms name. Difficulty adjusts to the student's performance. PLATO's help is gradually reduced until the student is finding the simplest terms name independently.

Equivalence Experience (touch)



- Purpose:
- 1) Encourage the student to think about why we can find the simplest terms name for a fraction by dividing out the common factors of the numerator and denominator.
 - 2) Give the student a chance to experiment with factors and divisibility related to equivalent fractions, using PLATO to do the computations.

Description: Some kids on the screen briefly explain that they can get another name for a fraction by dividing out common factors of the numerator and denominator. Then the student uses both multiplication and division to construct more names for given fractions. PLATO does the computation, so that the student is concerned only with exploring multiples, factors, divisibility, and any other patterns that he or she may find interesting.

Checkup: Simplest Terms

There is a short checkup sequence available in lesson "Simplest Terms" which provides review of the topic and an indication of whether the student has mastered the material.

EQUIVALENT FRACTIONS AND MIXED NUMBERS

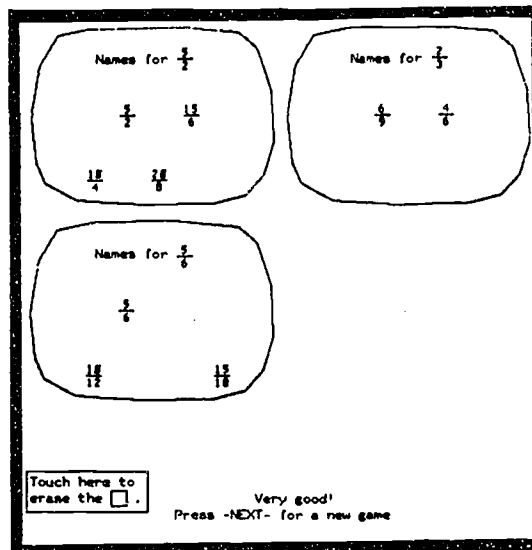
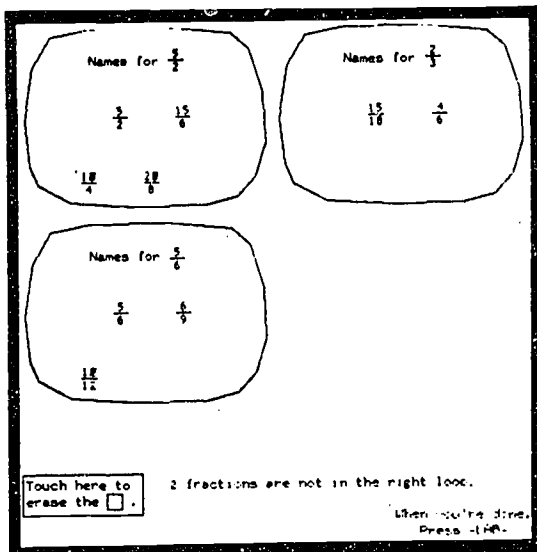
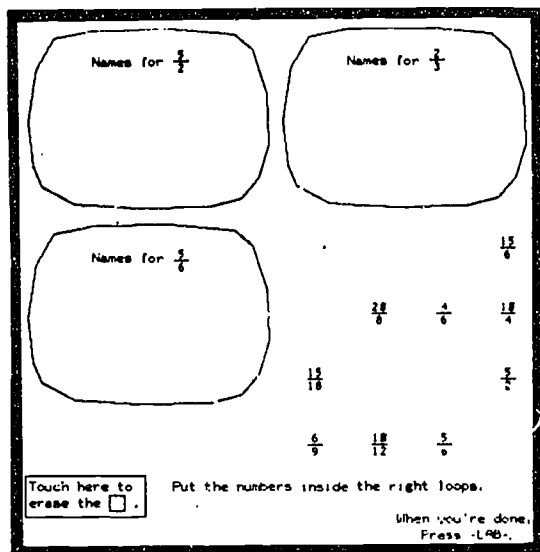
PRACTICE WITHOUT A MODEL

This set of lessons provides practice constructing and identifying fractions and mixed numbers that are equivalent to a given fraction or mixed number. The student is asked to sort fractions and mixed numbers so that all names for the same number are grouped together. Other exercises ask the student to complete statements like $1/2 = \underline{\quad}/6$.

Sort Equivalent Fractions	G3
Sort Equivalent Mixed Numbers	G4
Try These: Equivalent Fractions	G5
Checkups	G6

Note: Lessons that require the use of a touch panel are identified by the word "touch" after the title on the lesson description page.

Sort Equivalent Fractions (touch)



Purpose: Provide practice identifying equivalent fractions. This is for students who already have a basic understanding of equivalent fractions.

Description: Several fractions are scattered on the screen along with 2, 3, or 4 loops. The student sorts the fractions into the loops so that each loop contains an equivalence set. Difficulty adjusts to the student's performance.

Sort Equivalent Mixed Numbers (touch)

Names for 1

Names for $1\frac{3}{4}$

Touch here to erase the ☐ .

When you're done, Press -NEXT-.

Names for 1

Names for $1\frac{3}{4}$

Touch here to erase the ☐ .

Very good!
Press -NEXT- for a new game

Purpose: Provide practice identifying equivalent whole numbers, fractions, and mixed numbers. This is for students who already have a basic understanding of equivalent fractions and mixed numbers.

Description: Several numbers (whole numbers, fractions, and mixed numbers) are scattered on the screen along with 2, 3, or 4 loops. The student sorts the numbers into the loops so that each loop contains an equivalence set. Difficulty adjusts to the student's performance.

Try These: Equivalent Fractions

TRY THESE!

$\frac{1}{5} = \frac{2}{10}$
 $\frac{1}{3} = \frac{2}{6}$

$\frac{1}{4} = \frac{4}{16}$
 $\frac{1}{4} = \frac{1}{2}$

TRY THESE!

$\frac{1}{5} = \frac{3}{15}$ ok
 $\frac{1}{3} = \frac{2}{6}$ ok

$\frac{1}{3} = \frac{4}{12}$ no
 $\frac{2}{4} = \frac{1}{2}$ ok

$\frac{1}{4} = \frac{4}{16}$

Press I/E T.

Purpose: Provide practice with equivalent fractions. This is for students who have a basic understanding of equivalent fractions, but need practice.

Description: There are four problems per page, and no pictures or models. Problems are like $\frac{1}{2} = \frac{\quad}{6}$. If the student is wrong twice on a problem, PLATO gives the answer to it. Difficulty adjusts to the student's performance.

Checkups

TRY THESE!

$\frac{1}{3} = \frac{2}{6}$	$\frac{1}{4} = \frac{2}{8}$
$\frac{1}{2} = \frac{4}{8}$	$\frac{1}{4} = \frac{1}{2}$

<p>Names for $\frac{3}{2}$</p> <div style="border: 1px solid black; height: 80px; margin: 10px;"></div> <p>Names for $\frac{5}{6}$</p> <div style="border: 1px solid black; height: 80px; margin: 10px;"></div>	<p>Names for $\frac{2}{3}$</p> <div style="border: 1px solid black; height: 80px; margin: 10px;"></div>
---	--

Touch here to
erase the .

Put the numbers inside the right loops.

When you're done,
Press **LAB**.

Short checkup sequences are available in the following lessons:

Try These: Equivalent Fractions
Short Equivalent Fractions

These sequences serve to review the previous work on equivalent fractions and see if the student is ready for material that uses equivalent fractions.

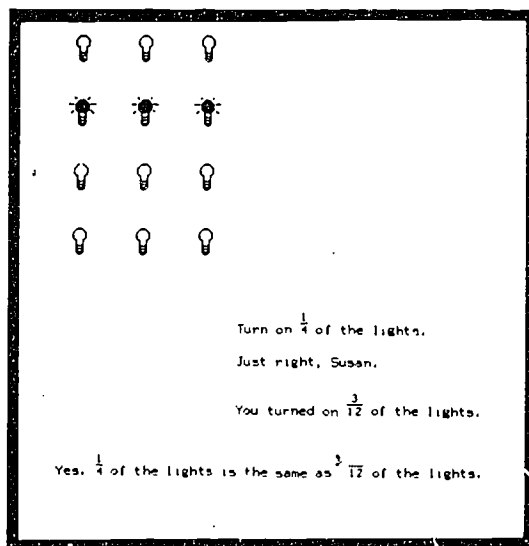
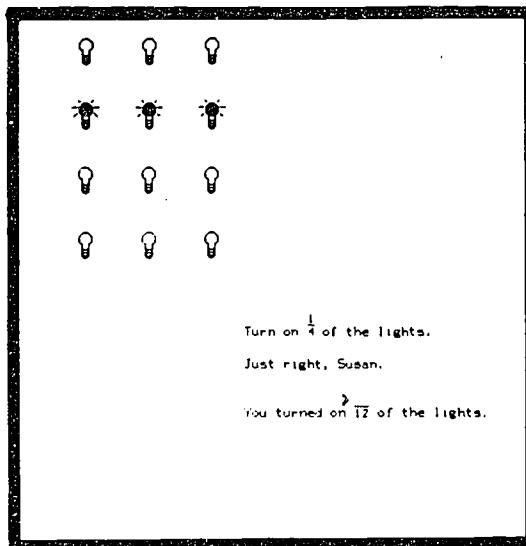
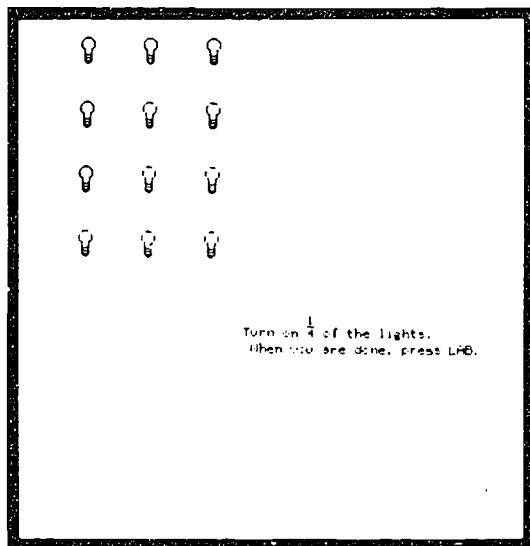
EXPLORATORY LESSONS FOR EQUIVALENT FRACTIONS AND COMMON DENOMINATORS

These lessons take an exploratory, intuitive approach to equivalent fractions and common denominators. The first lesson illustrates that two fractions can name the same part of a set of objects. The others provide experience using equivalent fractions and common denominators to solve problems like how to measure out half a cup of something when there is no $\frac{1}{2}$ cup measure available, or how to serve 3 people who want different fractions of a pizza when the pizza must be cut into some number of equal pieces.

Lights: Equivalent Fractions	H3
Pizza: Cutter Stuck	H4
Pizza: Common Denominator	H5
Make-a-Monster with Equivalent Fractions	H6
Make-a-Monster with One Cup	H7

Note: Lessons that require the use of a touch panel are identified by the word "touch" after the title on the lesson description page.

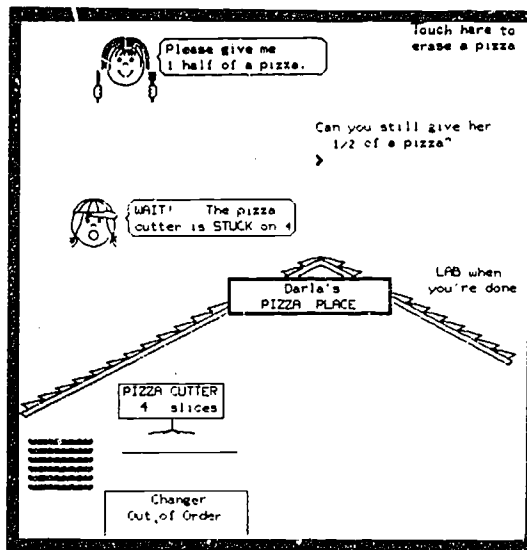
Lights: Equivalent Fractions (touch)



Purpose: Introduce equivalent fractions, using a set of objects.

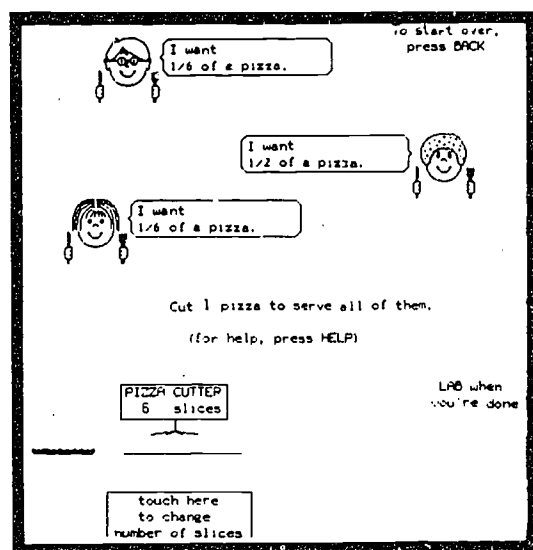
Description: The lesson consists of problems like this: The student turns on $\frac{1}{4}$ of 12 light bulbs (by turning on 1 row out of 4 rows), then notes that $\frac{3}{12}$ of the lights are on, and concludes that $\frac{1}{4}$ of the lights is the same as $\frac{3}{12}$ of the lights. Difficulty adjusts to the student's performance.

Pizza: Cutter Stuck (touch)



Purpose: Provide an informal introduction to equivalent fractions.

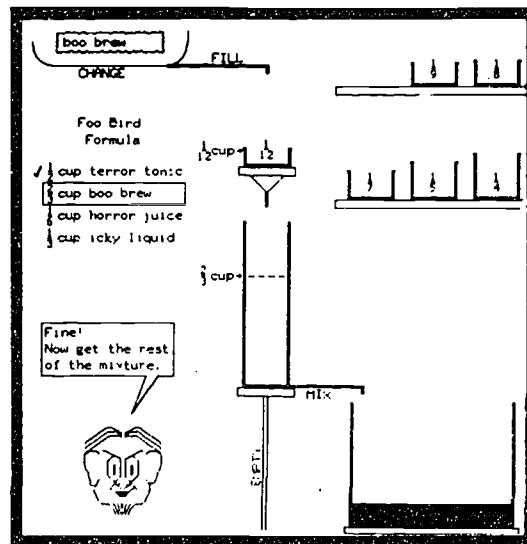
Description: The student operates pizza place, serving kids who ask for specific fractions of pizza. The automatic pizza cutter is stuck on the "wrong" number of slices, so, for example, the student has to use eighths to make a half.

Pizza: Common Denominator (touch)

Purpose: Provide experience finding and using a common denominator with a familiar model.

Description: The student has one pizza with which to serve two or three kids on the screen who want different fractions of pizza. The pizza must be cut once into equal pieces to serve everyone. Difficulty of problems adjusts to the student's performance.

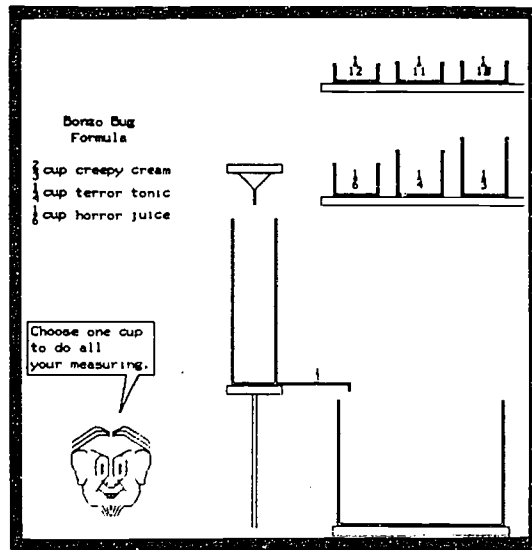
Make-a-Monster: Equivalent Fractions (touch)



Purpose: Provide experience using equivalent fractions with a familiar model.

Description: This is like the earlier Make-a-Monster, but the student has to use equivalent fractions. When the formula includes $\frac{3}{4}$ cup of something, the $\frac{1}{4}$ cup is not available, but the $\frac{1}{8}$ cup or $\frac{1}{12}$ cup is. The lesson records which cups a student has used for each fraction, and when possible, chooses available cups from those not yet used. If a student is doing particularly well, he or she may get "hard" formulas (e.g., a situation where he or she must measure $\frac{3}{9}$ with a $\frac{1}{6}$ cup).

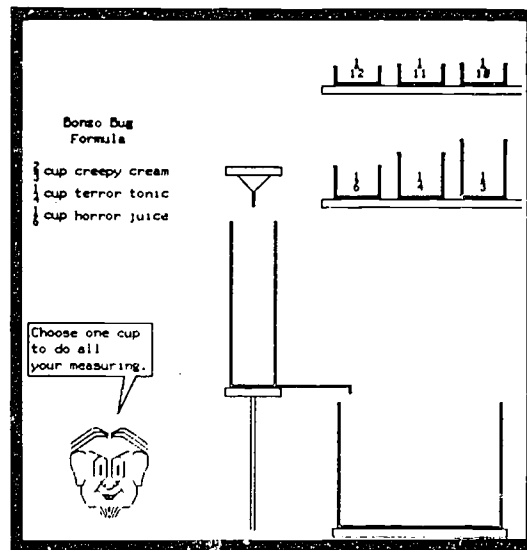
Make a Monster with One Cup (touch)



Purpose: Provide experience finding and using a common denominator with a familiar physical model.

Description: This is like the earlier Make-a-Monster, but the student must choose one measuring cup to do all the ingredients in the formula. For most students, this will be a straight forward common denominator exercise. But, for students who do particularly well in this program, there will be a few "hard" formulas. (A "hard" formula might include $\frac{6}{9}$ cup, $\frac{2}{4}$ cup, and $\frac{5}{5}$ cup; the appropriate cup choice would be $\frac{1}{6}$ or $\frac{1}{12}$.)

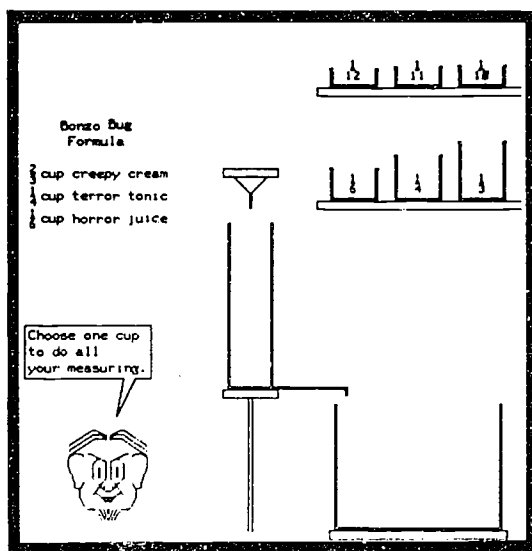
Make a Monster with One Cup (touch)



Purpose: Provide experience finding and using a common denominator with a familiar physical model.

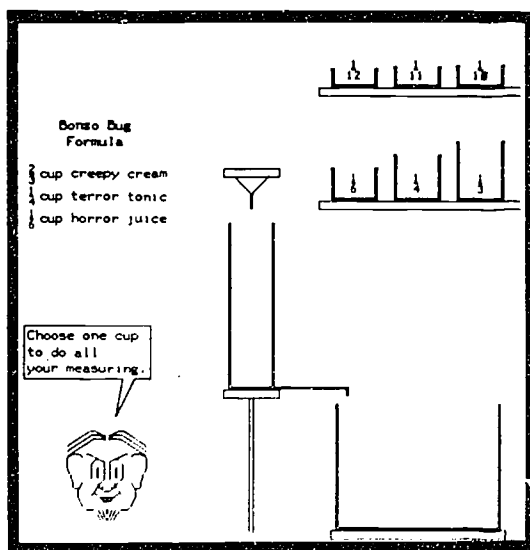
Description: This is like the earlier Make-a-Monster, but the student must choose one measuring cup to do all the ingredients in the formula. For most students, this will be a straight forward common denominator exercise. But, for students who do particularly well in this program, there will be a few "hard" formulas. (A "hard" formula might include $\frac{6}{9}$ cup, $\frac{2}{4}$ cup, and $\frac{5}{5}$ cup; the appropriate cup choice would be $\frac{1}{6}$ or $\frac{1}{12}$.)

Make a Monster with One Cup (touch)



Purpose: Provide experience finding and using a common denominator with a familiar physical model.

Description: This is like the earlier Make-a-Monster, but the student must choose one measuring cup to do all the ingredients in the formula. For most students, this will be a straight forward common denominator exercise. But, for students who do particularly well in this program, there will be a few "hard" formulas. (A "hard" formula might include $\frac{6}{9}$ cup, $\frac{2}{4}$ cup, and $\frac{5}{5}$ cup; the appropriate cup choice would be $\frac{1}{6}$ or $\frac{1}{12}$.)

Make a Monster with One Cup (touch)

Purpose: Provide experience finding and using a common denominator with a familiar physical model.

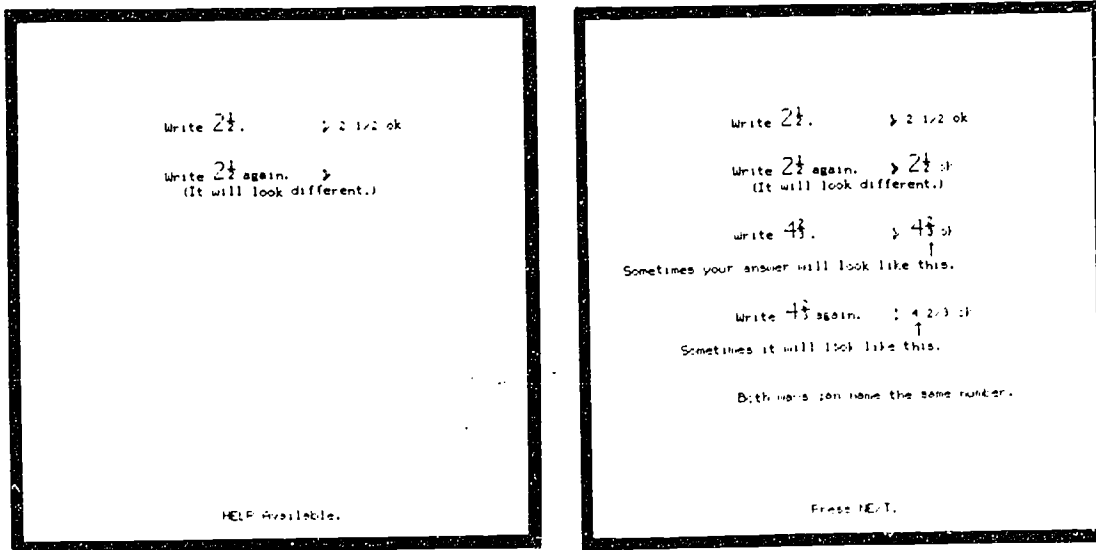
Description: This is like the earlier Make-a-Monster, but the student must choose one measuring cup to do all the ingredients in the formula. For most students, this will be a straight forward common denominator exercise. But, for students who do particularly well in this program, there will be a few "hard" formulas. (A "hard" formula might include $\frac{6}{9}$ cup, $\frac{2}{4}$ cup, and $\frac{5}{5}$ cup; the appropriate cup choice would be $\frac{1}{6}$ or $\frac{1}{12}$.)

ADDITION AND SUBTRACTION WITH LIKE DENOMINATORS

This set of lessons covers addition and subtraction of mixed numbers with like denominators. The instructional lessons use the pizza model to introduce the concepts of adding, subtracting, renaming an answer in standard form (i.e. with fraction part less than 1 and in simplest terms), and "borrowing" in subtraction. The pizza model is available for help through much of the practice. Some practice is done with number line.

New Answer Format	I3
Addition and Subtraction of Mixed Numbers with Like Denominators	I4
Simplifying Answers to Addition Problems	I5
Standard Form Practice	I6
Addition Practice, Simplifying Answers	I7
Subtraction with Borrowing	I8
Addition and Subtraction Practice with Simplifying and Borrowing	I9
Target Practice	I10
Fractions Basketball	I11
Addition and Subtraction Drill	I12
Checkups	I13

Note: Lessons that require the use of a touch panel are identified by the word "touch" after the title on the lesson description page.

New Answer Format

Purpose: Introduce the new way that a student's answer will look when he or she writes a fraction or mixed number on PLATO.

Description: The student writes several mixed numbers. Some of them appear as $3\frac{1}{2}$ (the "old" format); others appear as $3\frac{1}{2}$ (the "new" format).

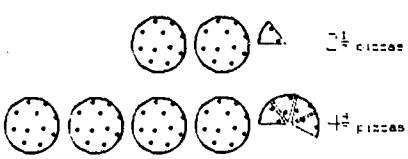
The new format will be used in the addition and subtraction lessons where the old format might look confusing. The new format involves no change in the keys pressed by the student, only in the way PLATO shows it. The old format is easier for PLATO to handle and allows many more options for the student, so it will continue to be used whenever the new format is not needed.

Addition and Subtraction of Mixed Numbers with Like Denominators (touch)

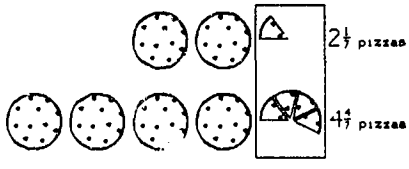
HELP available

$$\begin{array}{r} 2\frac{1}{2} \\ + 4\frac{1}{2} \\ \hline \end{array}$$

>



How many pizzas in all?


$$\begin{array}{r} 2\frac{1}{2} \\ + 4\frac{1}{2} \\ \hline \end{array}$$


How many whole pizzas? 6 ok
How many sevenths of a pizza? >

HELP available

$$\begin{array}{r} 2\frac{1}{2} \\ - 1\frac{1}{2} \\ \hline \end{array}$$

>

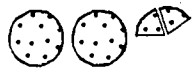


If you took away $1\frac{1}{2}$ pizzas, how much would you have left?

LFB when you're done. To start over, press BACK.

$$\begin{array}{r} 2\frac{1}{2} \\ - 1\frac{1}{2} \\ \hline \end{array}$$

Hello, Barry. How many pizzas I have $1\frac{1}{2}$ pizzas.

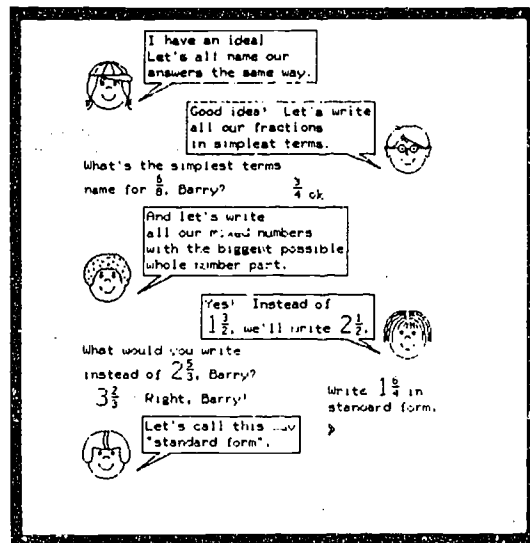
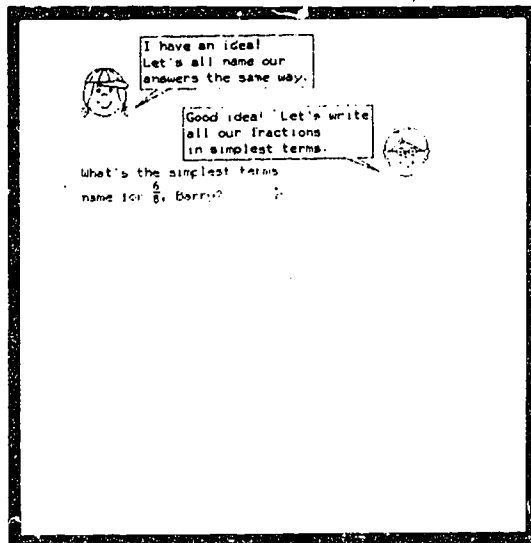
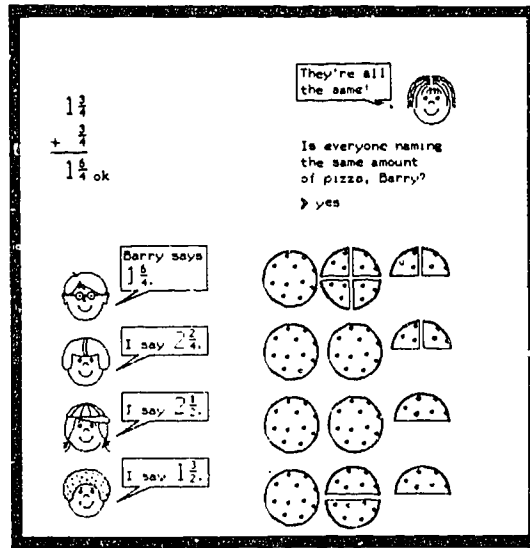
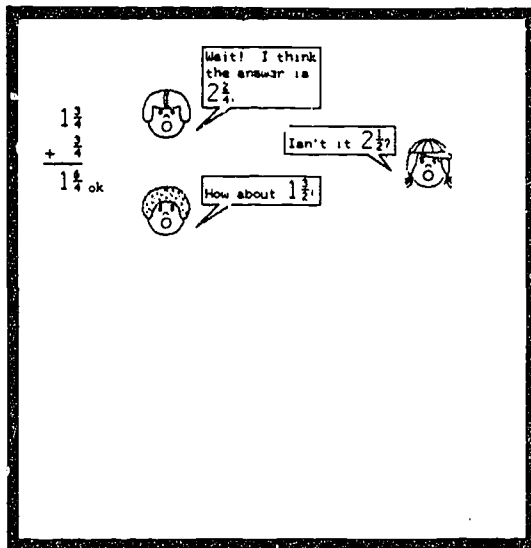


You are starting with $2\frac{1}{2}$ pizzas.

Purpose: Begin addition and subtraction of fractions and mixed numbers with like denominators.

Description: The student answers addition and subtraction problems. Pizza pictures and extra help are available. Difficulty adjusts to the student's performance. The student who is doing well gets some problems without pictures.

Simplifying Answers to Addition Problems



Purpose: Show a need for simplifying the answer to an addition problem.

Description: The student answers an addition problem. Several kids on the screen suggest different correct answers to the same problem. In order to avoid this confusion, they decide to always name their answers with the biggest possible whole number and with the fraction in simplest terms. They call this way "standard form." The student practices writing some numbers in standard form.

Standard Form Practice

Standard form must use
biggest whole number possible
and fraction in simplest terms.

Write the standard form name for these numbers:

$\frac{6}{8} = \frac{3}{4}$ ok $\frac{212}{18} = 3\frac{7}{9}$ ok

$3\frac{6}{8} = 3\frac{3}{4}$ no $4\frac{5}{7}$ ok

That's a name for $3\frac{3}{4}$,
but we want the
biggest possible
whole number.

Purpose: Practice writing fractions and mixed numbers in standard form.

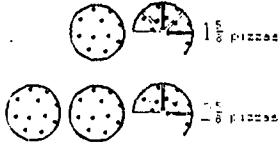
Description: The student writes the standard form names for given fractions and mixed numbers. There are four problems per page. After the student's third wrong try on a problem, PLATO gives the answer. Difficulty adjusts to the student's performance.

Addition Practice, Simplifying Answers

HELP
available

$$\begin{array}{r} 1\frac{5}{8} \\ + 2\frac{5}{8} \\ \hline 3\frac{10}{8} \rightarrow \end{array}$$

Right number, Barry.
Name it in
standard form.

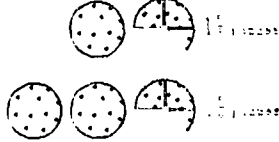


How many pizzas in all?

HELP
available

$$\begin{array}{r} 1\frac{5}{8} \\ + 2\frac{5}{8} \\ \hline 3\frac{10}{8} = 4\frac{1}{2} \end{array}$$

Good, but put
the fraction part
in simplest terms.

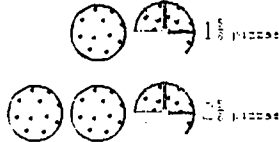


How many pizzas in all?

HELP
available

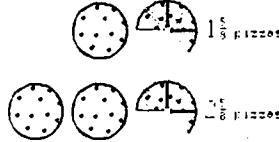
$$\begin{array}{r} 1\frac{5}{8} \\ + 2\frac{5}{8} \\ \hline 3\frac{10}{8} \rightarrow 3\frac{5}{4} \end{array}$$

Still the right number, Barry, but it's
not in standard form. Put together some of
those pieces to make a whole pizza.



How many pizzas in all?

HELP
available

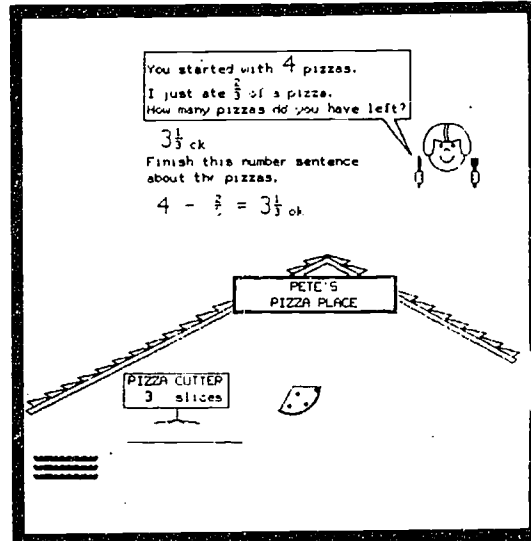
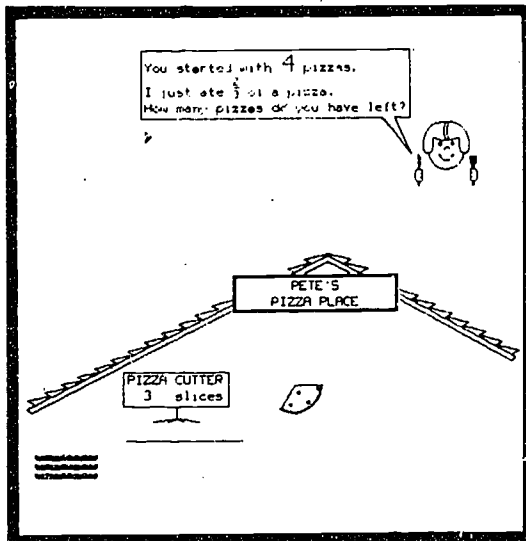
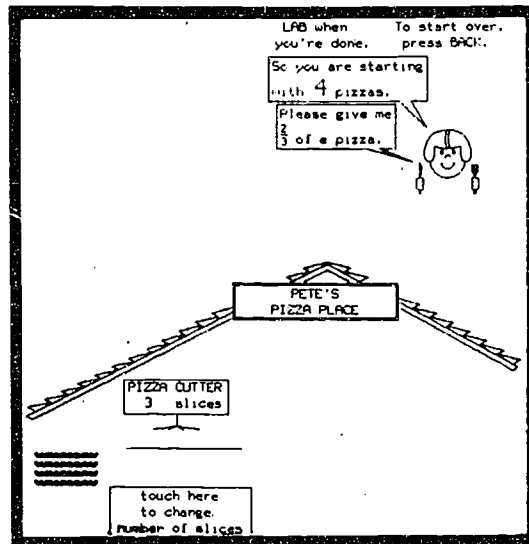
$$\begin{array}{r} 1\frac{5}{8} \\ + 2\frac{5}{8} \\ \hline 3\frac{10}{8} = 4\frac{1}{2} \end{array}$$


How many pizzas in all?

Purpose: Provide experience adding fractions and mixed numbers with like denominators, simplifying answers when necessary.

Description: The student answers addition problems, simplifying answers when necessary. Pizza pictures and extra help are available. Difficulty adjusts to the student's performance.


Subtraction with Borrowing (touch)



Purpose: Introduce a method for solving subtraction problems like $3 \frac{1}{3} - \frac{2}{3}$.

Description: The student does "borrowing" problems while operating a pizza place. For example, the student starts with $3 \frac{1}{3}$ pizzas, then gives away $\frac{2}{3}$ of a pizza by cutting one of the whole pizzas. The student writes a subtraction number sentence: $3 \frac{1}{3} - \frac{2}{3} = 2 \frac{2}{3}$. Difficulty adjusts to the student's performance. Harder problems have less specific step-through and ask the student to write the subtraction number sentence before cutting the pizza.


Addition and Subtraction Practice with Simplifying and Borrowing



Two whole pizzas and one-fifth of a pizza are shown. Below them is a subtraction problem:

$$\begin{array}{r} 2\frac{1}{5} \\ - 1\frac{3}{5} \\ \hline \end{array}$$

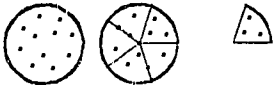
An arrow points from the first pizza to the second with the text "Cut 1 whole pizza into fifths."



One whole pizza, one pizza cut into fifths, and one-fifth of a pizza are shown. Below them is a subtraction problem:

$$\begin{array}{r} 2\frac{1}{5} \\ - 1\frac{3}{5} \\ \hline \end{array}$$

An arrow points from the first pizza to the second with the text "Cut 1 whole pizza into fifths." The result of the subtraction is shown as $1\frac{2}{5}$.



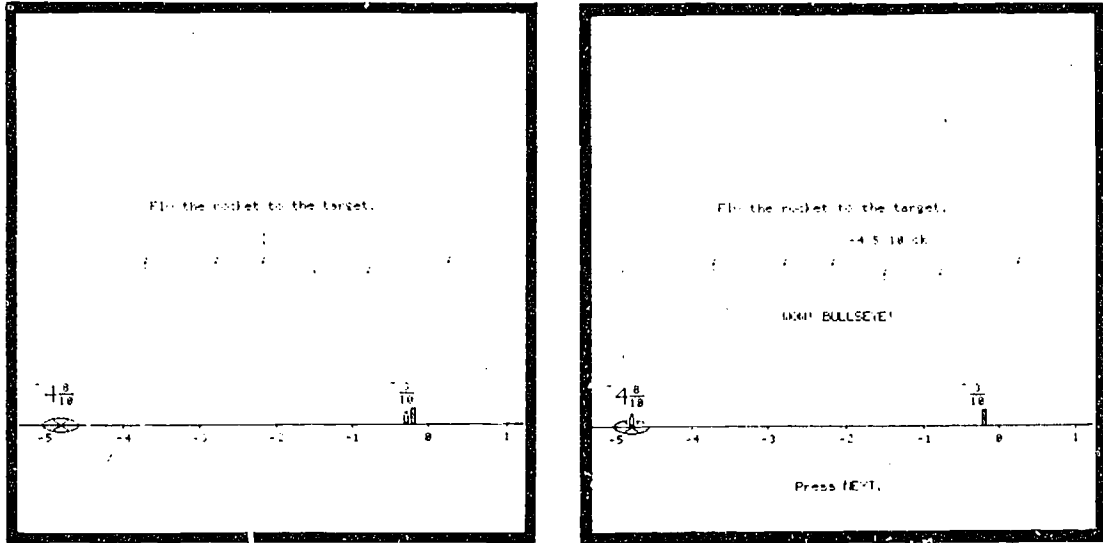
One whole pizza, one pizza cut into fifths, and one-fifth of a pizza are shown. Below them is a subtraction problem:

$$\begin{array}{r} 2\frac{1}{5} \\ - 1\frac{3}{5} \\ \hline \end{array}$$

An arrow points from the first pizza to the second with the text "Cut 1 whole pizza into fifths." The result of the subtraction is shown as $1\frac{2}{5}$.

Purpose: Provide experience adding and subtracting fractions and mixed numbers, simplifying answers and "borrowing" when necessary.

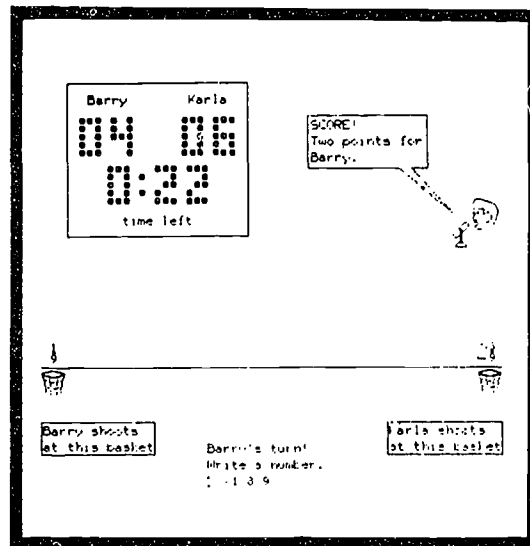
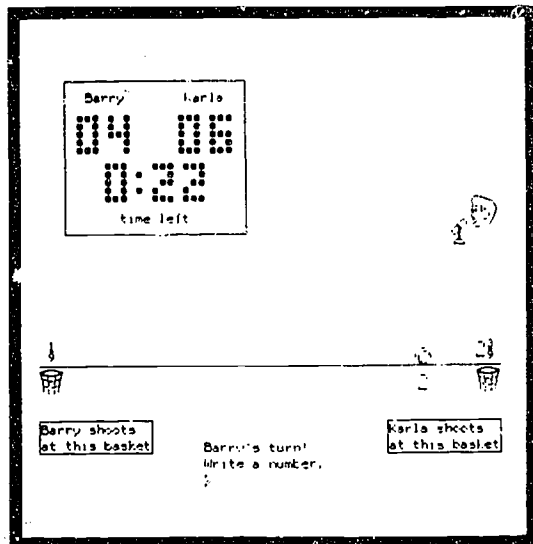
Description: The student is stepped through subtraction problems involving "borrowing." Difficulty adjusts to the student's performance. Higher levels of difficulty give less help and include addition problems as well as subtraction problems. Most, but not all, of the subtraction problems involve "borrowing."

Target Practice

Purpose: Addition practice which may encourage checking the answer by estimation.

Description: The student fires a remote control rocket at a target. Both the rocket and the target are on a number line at known positions. An easy estimate will hit the target for a "SCORE," but only the exact answer gets a "BULLSEYE." Difficulty adjusts to the student's performance.

Fractions Basketball



Purpose: Subtraction practice on the number line.

Description: This is a game for one or two students. Given ball and basket positions on the number line, the student bounces the ball some fraction or mixed number (e.g., $2 \frac{7}{8}$ makes the ball do two bounces of length 1 then seven bounces of length $\frac{1}{8}$) to make a basket. Difficulty adjusts to the student's performance.

Addition and Subtraction Drill

$2\frac{2}{18} + 3 \rightarrow$	$\frac{1}{18} + 1 =$
$\frac{4}{7} - \frac{1}{7} =$	$\frac{4}{9} + \frac{1}{9}$

$2\frac{2}{18} + 3 = 5\frac{2}{18} \text{ ok}$	$\frac{1}{18} + 1 = 1\frac{1}{18} \text{ ok}$
<p>Did you add instead of subtract?</p> $\frac{4}{7} - \frac{1}{7} \rightarrow \frac{5}{7}$	$\frac{4}{9} + \frac{1}{9}$

Purpose: Practice adding and subtracting fractions and mixed numbers with like denominators. This is for students who have a basic understanding but need practice.

Description: There are four problems per page and no pictures or models. There is help for students who have trouble "borrowing" or putting the answer in standard form. There are special messages for common errors like adding instead of subtracting. Difficulty adjusts to the student's performance.

my my
77

Checkups

Standard form name uses
biggest whole number possible
and fraction in simplest terms.

Write the standard form name for these numbers:

$\frac{6}{8} = \frac{3}{4}$ $\frac{17}{10} = 1\frac{7}{10}$

$3\frac{6}{8} = 3\frac{3}{4}$ $4\frac{1}{2}$

That's a name for $3\frac{3}{4}$,
but we want the
biggest possible
whole number.

Do the problem on paper or in your head.

$$\begin{array}{r} 2\frac{1}{2} \\ - 1\frac{1}{2} \\ \hline \end{array}$$

Put your answer here.

Short checkup sequences are available in the following lessons:

Standard Form Practice

Addition and Subtraction Practice with Simplifying and Borrowing

These sequences serve to review the previous work on addition of mixed numbers and see if the student is ready for more advanced material.

ADDITION AND SUBTRACTION WITH UNLIKE DENOMINATORS --

A HEURISTICS APPROACH

This set of lessons uses a heuristics approach to teach addition and subtraction of mixed numbers with unlike denominators. It is assumed that the student can already add and subtract mixed numbers with like denominators. The method used in these lessons is to first find out what makes the problem hard (it has unlike denominators), then find an easy problem (one that has like denominators) that has the same answer. Problems with unlike denominators are renamed with like denominators.

Sort Equivalent Problems	J3
Addition by Using an Easier Problem:	
Part 1	J4
Addition by Using an Easier Problem:	
Part 2	J5
Addition by Using an Easier Problem:	
Part 3	J6
Addition by Using an Easier Problem:	
Part 3 Practice	J7
Addition by Using an Easier Problem:	
Part 4	J8
Addition by Using an Easier Problem:	
Part 4 Practice	J9
Addition by Using an Easier Problem:	
Review	J10
Addition and Subtraction Drill	J11

Note: Lessons that require the use of a touch panel are identified by the word "touch" after the title on the lesson description page.

Sort Equivalent Problems (touch)

The image shows two screenshots of a game interface. Each screenshot has a black border and contains two rounded rectangular boxes at the top labeled 'NAMES FOR'. Below these boxes are several subtraction problems involving fractions. At the bottom of each screen is an 'erase' button and instructional text.

Left Screenshot:

- Left box: NAMES FOR $\frac{1}{4} - 2\frac{1}{2}$
- Right box: NAMES FOR $\frac{1}{6} - 2\frac{1}{2}$
- Problems on screen: $\frac{3}{18} - 2\frac{1}{2}$, $\frac{3}{12} - 2\frac{1}{2}$, $\frac{1}{4} - 2\frac{3}{4}$, $\frac{1}{6} - 2\frac{3}{8}$, $\frac{1}{6} - 2\frac{3}{4}$, $\frac{2}{6} - 2\frac{1}{2}$
- Buttons: 'Touch here to erase the' with a small square icon.
- Text: 'Put the problems inside the right loops. When you're done, Press -L&R-'

Right Screenshot:

- Left box: NAMES FOR $\frac{1}{4} - 2\frac{1}{2}$
- Right box: NAMES FOR $\frac{1}{6} - 2\frac{1}{2}$
- Problems on screen: $\frac{1}{4} - 2\frac{3}{4}$, $\frac{2}{6} - 2\frac{1}{2}$, $\frac{3}{12} - 2\frac{1}{2}$, $\frac{1}{6} - 2\frac{3}{4}$, $\frac{1}{6} - 2\frac{3}{8}$
- Buttons: 'Touch here to erase the' with a small square icon.
- Text: 'Very good! Press -NEXT- for a new game.'

Purpose: Introduce the idea that renaming the numbers in an addition or subtraction problem makes another name for the same problem. This idea will be useful when we begin finding a common denominator for addition and subtraction.

Description: Several expressions involving addition and subtraction are scattered on the screen with two or three loops to sort them into. The student sorts the expressions into the loops so that each loop contains an equivalence set. Difficulty adjusts to the student's performance.

Addition by Using an Easier Problem: Part 1

$2\frac{2}{3} - 1\frac{1}{3} = 1\frac{1}{3}$ ok

$1\frac{2}{3}$
+ $\frac{1}{3}$
1 $\frac{3}{3}$ ok

"All the old problems are like this one. It's all thirds!"

We have a new kind of problem.

$1\frac{2}{3}$
+ $4\frac{1}{6}$

We have to add thirds and sixths together!

I see. Different denominators make the new one hard.

$2\frac{2}{3} - 1\frac{1}{3} = 1\frac{1}{3}$ ok

$1\frac{2}{3}$
+ $\frac{1}{3}$
1 $\frac{3}{3}$ ok

We need names for $1\frac{2}{3}$ and $4\frac{1}{6}$ that have the same denominators.

$1\frac{4}{6}$ $1\frac{2}{3}$ is a name for $1\frac{2}{3}$.

+ $4\frac{1}{6}$

Try it, Helen.

Let's name them both with sixths.

$2\frac{2}{3} - 1\frac{1}{3} = 1\frac{1}{3}$ ok

$1\frac{2}{3}$
+ $\frac{1}{3}$
1 $\frac{3}{3}$ ok

Uhew! Thanks, Helen!

$1\frac{4}{6}$ $1\frac{4}{6}$ is a name for $1\frac{2}{3}$.
+ $4\frac{1}{6}$ $+ 4\frac{1}{6}$ is a name for $4\frac{1}{6}$.
 $5\frac{5}{6}$ ok

But what about the hard problem?

$2\frac{2}{3} - 1\frac{1}{3} = 1\frac{1}{3}$ ok

$1\frac{2}{3}$
+ $\frac{1}{3}$
1 $\frac{3}{3}$ ok

It has the same answer as the easy one.

$1\frac{4}{6}$ $1\frac{4}{6}$ is a name for $1\frac{2}{3}$.
+ $4\frac{1}{6}$ $+ 4\frac{1}{6}$ is a name for $4\frac{1}{6}$.
 $5\frac{5}{6}$ ok

Because they're really the same problem - just with different names.

I see! So now we have the answer to the hard problem! Write it for us, Helen.

- Purpose: 1) Introduce addition of fractions and mixed numbers with unlike denominators.
2) Introduce an approach to problem-solving.

Description: The student answers 2 problems with like denominators, then PLATO presents one with unlike denominators. Kids on the screen discuss what makes the new "hard" problem different from the similar "easy" ones. They decide that by renaming the fractions in the "hard" problem, they can find an "easy" problem that has the same answer as the "hard" one. The student fills in the numbers as the kids on the screen step through the process. This is a brief introduction and overview. Subsequent lessons develop these ideas more fully.

Addition by Using an Easier Problem: Part 2

$$\begin{array}{r} 2 \text{ fifths} \\ + 1 \text{ fifth} \\ \hline 1 \text{ fifth ok} \end{array}$$

That's an easy problem!

It's all fifths.

$$\begin{array}{r} 3 \text{ tenths} \\ + 2 \text{ fifths} \end{array}$$

But this one is hard!

Too bad it's not all tenths or all fifths.

Can we find an easy problem that has the same answer?

How about this one? It's easy.

$$\begin{array}{r} 3 \text{ tenths} \\ + 2 \text{ fifths} \end{array}$$

$$\begin{array}{r} 3 \text{ tenths} \\ + 2 \text{ tenths} \end{array}$$

It's easy, but does it have the same answer as the hard one?

I don't know. Does it Helen?

What about this one? It's easy!

$$\begin{array}{r} 3 \text{ tenths} \\ + 2 \text{ fifths} \end{array}$$

$$\begin{array}{r} 3 \text{ tenths} \\ + 4 \text{ tenths} \end{array}$$

It's easy, but does it have the same answer as the hard one?

I don't know. Does it Helen? yes.

2 fifths and 4 tenths name the same number.

$$\begin{array}{r} 3 \text{ tenths} \\ + 2 \text{ fifths} \\ \hline \end{array}$$

$$\begin{array}{r} 3 \text{ tenths} \\ + 4 \text{ tenths} \\ \hline 7 \text{ tenths ok} \end{array}$$

The hard problem has the same answer as the easy one!

Write the answer for us, Helen.

Purpose: Give the student a more detailed look at the problem-solving approach to addition of fractions with unlike denominators.

Description: The student answers the problem $2 \text{ fifths} + 1 \text{ fifth}$. PLATO presents a new problem, $3 \text{ tenths} + 2 \text{ fifths}$. Kids on the screen discuss what makes the first problem "easy" and the second one "hard." They look for an "easy" problem that has the same answer as the "hard" one. They find the "easy" problem they need, but in the process they suggest 1 or 2 "easy" problems that won't do. Thus they emphasize that the appropriate "easy" problem must be another name for the "hard" one. The student answers questions and fills in numbers to help the kids on the screen.

Addition by Using an Easier Problem: Part 3

Right! It's 15 cents.

1 dime 1 nickel

How much money altogether? 15 cents

Why is that?

Right! It's 15 cents.

Because 1 nickel is 5 cents, and 1 dime is 10 cents.

1 dime 1 nickel

How much money altogether? 15 cents

So, 1 nickel + 1 dime is the same as 5 cents + 10 cents.

I see! 5 cents + 10 cents is another name for 1 nickel + 1 dime.

Try this one, Helen!

1 quarter → 25 cents
- 2 dimes

Be careful!
It's subtraction!

Try this one, Helen!

1 quarter → 25 cents
- 2 dimes → 20 cents
5 cents

Be careful!
It's subtraction!

- Purpose: 1) Show addition of coins as a familiar process which is similar to addition of fractions with unlike denominators.
2) Reinforce the problem-solving approach to addition of fractions with unlike denominators.

Description: PLATO shows 1 nickel and 1 dime. The student tells how much money that is all together. Kids on the screen comment that "5 cents + 10 cents" is another name for "1 nickel + 1 dime." The student then does two or three coin addition problems by renaming each one in cents to make an "easy" problem that has the same answer as the "hard" one.

Addition by Using an Easier Problem: Part 3 Practice

Can you do these?

$$\frac{1}{4} \rightarrow \frac{2}{8}$$

$$+ \frac{5}{8} \rightarrow + \frac{5}{8}$$

$$\frac{7}{8} \text{ ok} \quad \frac{7}{8} \text{ ok}$$

$$\frac{1}{6} \rightarrow \frac{1}{6}$$

$$+ \frac{2}{3} \rightarrow + \frac{4}{6}$$

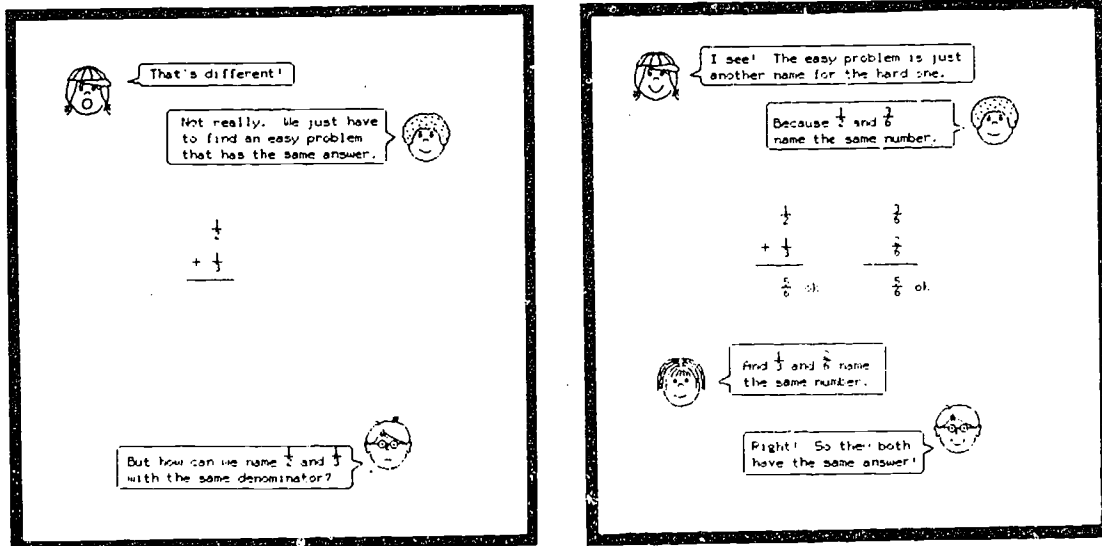
$$\frac{5}{6} \text{ ok} \quad \frac{5}{6} \text{ ok}$$

2 dimes \rightarrow 20 cents
 + 3 nickels \rightarrow + 15 cents
 35 cents

Purpose: Experience using the problem-solving approach to addition and subtraction of fractions with unlike denominators.

Description: The student solves addition and subtraction problems involving coins and fractions with unlike denominators. Difficulty adjusts to the student's performance. The lowest difficulty level is mostly coins problems. Higher levels have fewer coins problems, more fractions problems, and fractions written in fraction notation ("2/3" instead of "2 thirds"). The highest level has no coins problems and encourages the student to find a common denominator before PLATO suggests one. For fractions problems on all levels, one of the denominators in the problem can be used as the common denominator (i.e., one denominator always divides the other evenly).

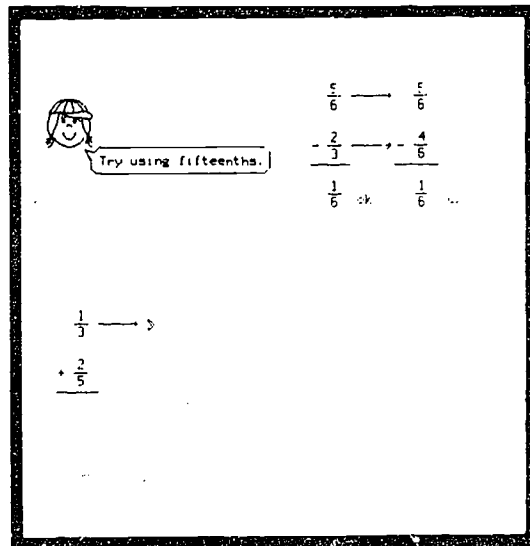
Addition by Using an Easier Problem, Part 4



- Purpose:
- 1) Review the problem-solving strategy for addition and subtraction of fractions with unlike denominators.
 - 2) Apply this approach to problems in which neither of the given denominators can be used for the common denominators (i.e., neither denominator divides the other evenly).

Description: The student starts with a page from "Addition by Using an Easier Problem: Part 3 Practice." If the student performs poorly on this page, he or she stays in the practice lesson. If the student does reasonably well, PLATO presents the problem, $\frac{1}{2} + \frac{1}{3}$. Kids on the screen quickly review the problem-solving strategy and suggest renaming the problem in sixths. The student works the problem, then the kids on the screen review the idea that the "easy" problem is just another name for the "hard" one.


Addition by Using an Easier Problem: Part 4 Practice



Purpose: Practice adding and subtracting fractions with unlike denominators. PLATO suggests a common denominator.

Description: The student solves addition and subtraction problems with unlike denominators. All problems use fraction notation ("2/3" instead of "2 thirds"). Difficulty adjusts to the student's performance. Higher levels have more problems in which neither of the denominators can be used as the common denominator (i.e., neither denominator divides the other evenly). The highest level encourages the student to find a common denominator before PLATO suggests one.



Addition by Using an Easier Problem: Review



First find out what makes it hard.

$$\begin{array}{r} 1\frac{1}{6} \\ + 1\frac{1}{3} \\ \hline \end{array}$$

It's hard because the denominators are not alike.

First find out what makes it hard.


Then find an easy problem that has the same answer.

$$\begin{array}{r} 1\frac{1}{6} \\ + 1\frac{1}{3} \\ \hline \end{array}$$

$$\begin{array}{r} 1\frac{1}{6} \\ + 1\frac{1}{3} \\ \hline \end{array}$$

Here's an easy problem that has the same answer.

Write the answer, Helen.



Purpose: Summarize and review the strategy for adding and subtracting fractions with unlike denominators.

Description: There are 6 problems involving addition and subtraction of fractions and mixed numbers with unlike denominators. For each problem, the kids on the screen review the problem-solving strategy and find an "easy" problem that has the same answer as the "hard" one. The student answers the problems. In a given session the student will see 1 or 2 of the problems.

Addition and Subtraction Drill

$$\frac{1}{2} + 2\frac{1}{4} = 2\frac{3}{4} \text{ ok}$$

$$\begin{array}{r} 2 \\ + \frac{1}{4} \\ \hline 2\frac{1}{4} \text{ ok} \end{array}$$

$$\begin{array}{r} \frac{1}{6} \rightarrow \frac{1}{6} \\ + \frac{1}{3} \rightarrow + \frac{2}{6} \\ \hline \frac{3}{6} \end{array}$$

$$\begin{array}{r} \frac{1}{18} \\ + \frac{1}{18} \\ \hline \frac{2}{18} \end{array}$$

$$\frac{1}{2} + 2\frac{1}{4} = 2\frac{3}{4} \text{ ok}$$

$$\begin{array}{r} 2 \\ + \frac{1}{4} \\ \hline 2\frac{1}{4} \text{ ok} \end{array}$$

$$\begin{array}{r} \frac{1}{6} \rightarrow \frac{1}{6} \\ + \frac{1}{3} \rightarrow + \frac{2}{6} \\ \hline \frac{3}{6} \text{ ok} \end{array}$$

$$\begin{array}{r} \frac{1}{18} \\ + \frac{1}{18} \\ \hline \frac{2}{18} \end{array}$$

Right number, Tom.
Now write it with the
fraction in simplest terms.

Purpose: Practice adding and subtracting fractions and mixed numbers with unlike denominators. This is for students who have a basic understanding but need practice.

Description: There are 4 problems per page and no pictures or models. There is help for students who have trouble re-naming with a common denominator, "borrowing," or putting answers into standard form. There are special messages for common errors like adding instead of subtracting. Difficulty adjusts to the student's performance.

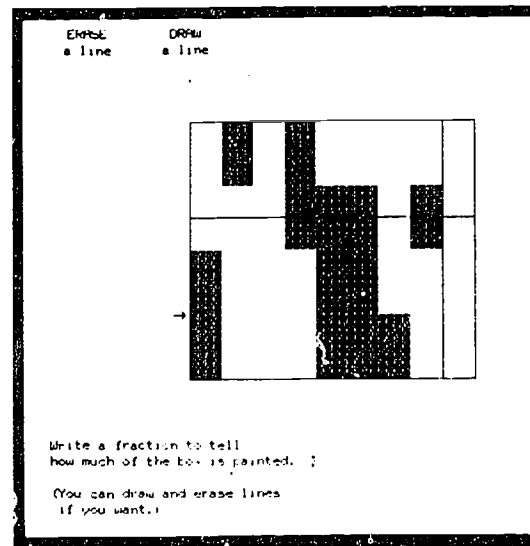
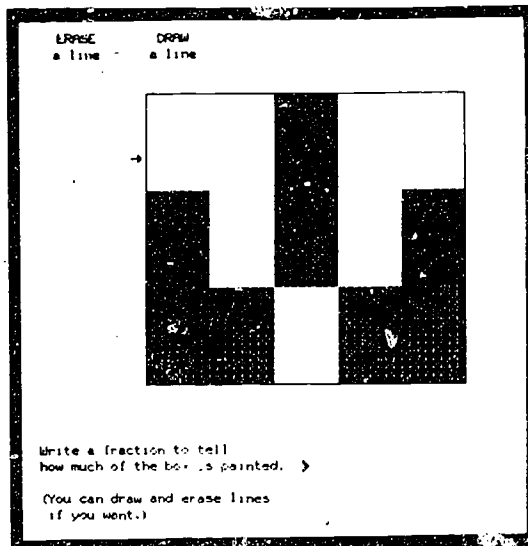
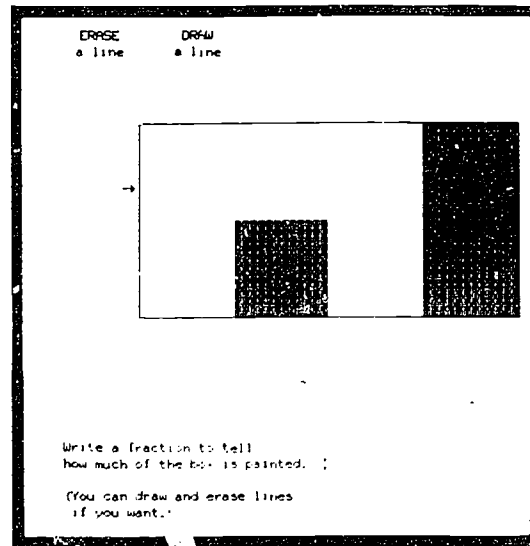
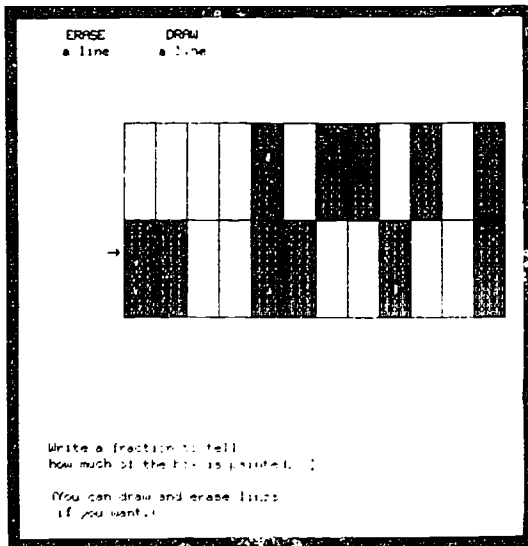
ADDITION WITH UNLIKE DENOMINATORS -- A VISUAL APPROACH

This set of lessons uses visual models to present addition of fractions with unlike denominators. The primary model is a rectangular box which is painted with two different kinds of paint. The task is to tell what fraction of the box is painted with each kind of paint, then what fraction of the box is painted in all. To tell what fraction is painted in all, the student is expected to draw lines dividing the box into equal pieces that are either painted or unpainted. This is a visual method of finding a common denominator. A number line is also used to give a visual interpretation of addition with a common denominator as well as general practice.

Boxes: How Much is Painted?	K3
Paint Addition: PLATO Paints	K4
Paint Addition: You Paint	K5
Paint Addition: Your Choice	K6
High Wire	K7
Target Practice	K8
Fractions Basketball	K9
Checkup: Paint Addition	K10

Note: Lessons that require the use of a touch panel are identified by the word "touch" after the title on the lesson description page.

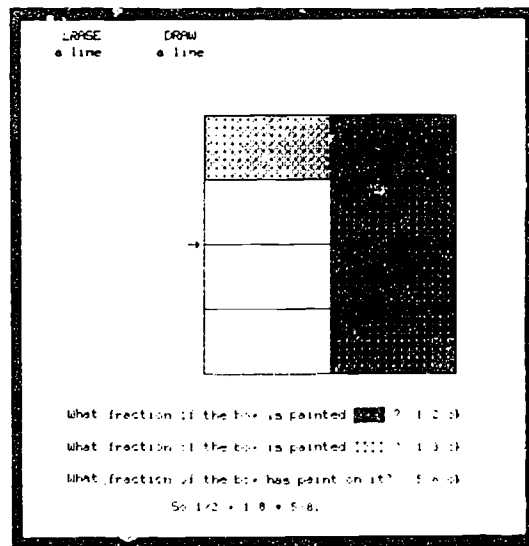
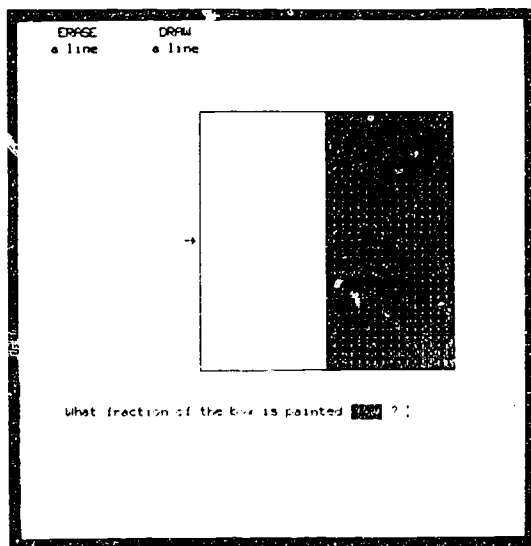
Boxes: How Much Is Painted? (touch)



Purpose: Develop a strategy for figuring out what fraction of a box is painted. This skill will be useful for addition of fractions using painted areas of a box.

Description: PLATO paints part of a box. The student can draw and erase lines to figure out what fraction of the box is painted. Difficulty adjusts to the student's performance.

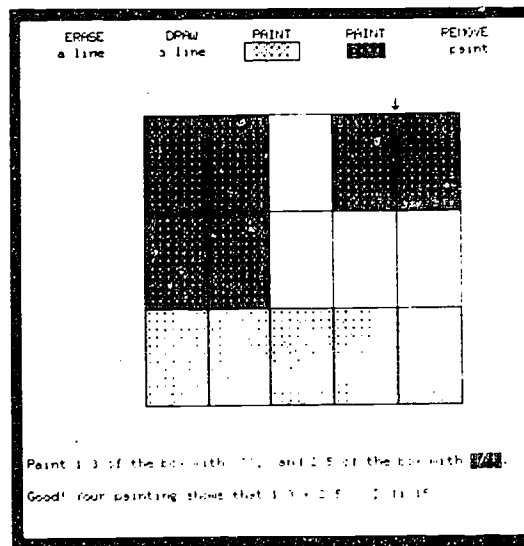
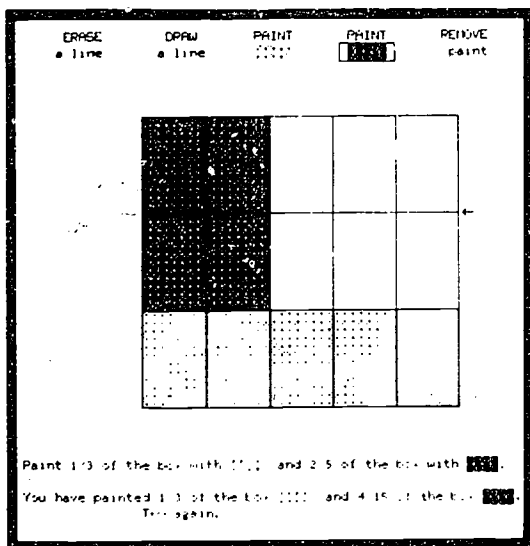
Paint Addition: PLATO Paint: (touch)



Purpose: Begin addition of fractions with unlike denominators, using a familiar model.

Description: PLATO uses 2 different kinds of paint to paint 2 fractions of the same box. The student tells what fraction of the box is painted with each kind of paint, then what fraction of the box is painted in all. The student can draw and erase lines on the box. This lesson relies on the student's experience cutting a box into sufficiently small equal pieces to tell how much is painted. Difficulty adjusts to the student's performance.

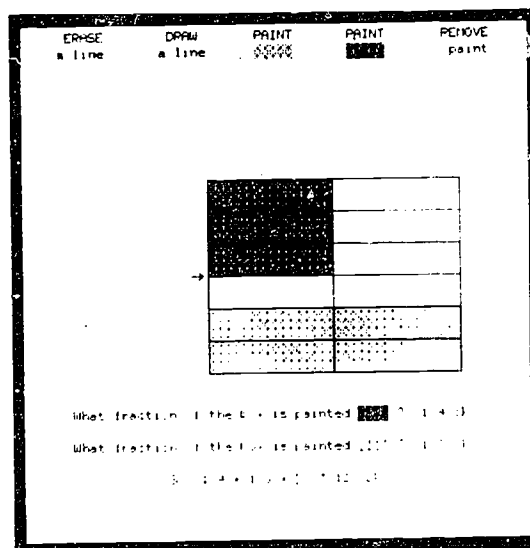
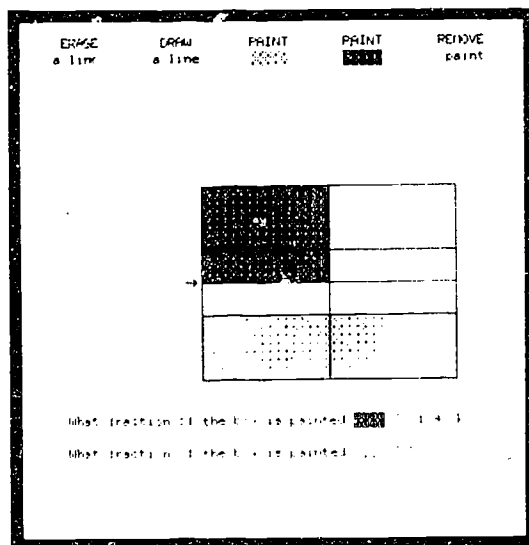
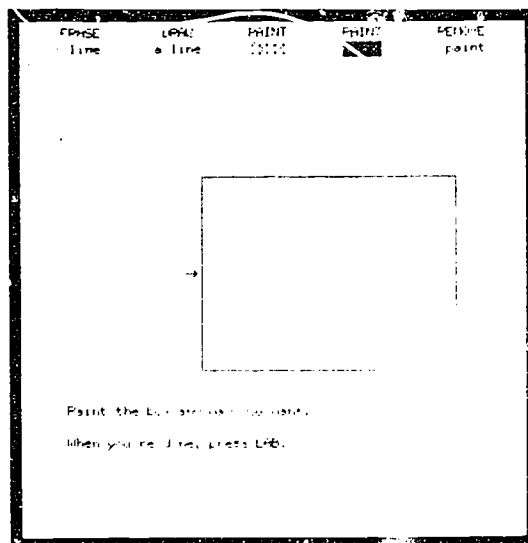
Paint Addition: You Paint (touch)



Purpose: Provide further experience with addition of fractions with unlike denominators, using a familiar model.

Description: The student is asked to paint a fraction of a box with one kind of paint, then paint another fraction of the box with a different kind of paint. The student then tells what fraction of the box has paint on it. Difficulty adjusts to the student's performance.

Paint Addition: Your Choice (touch)



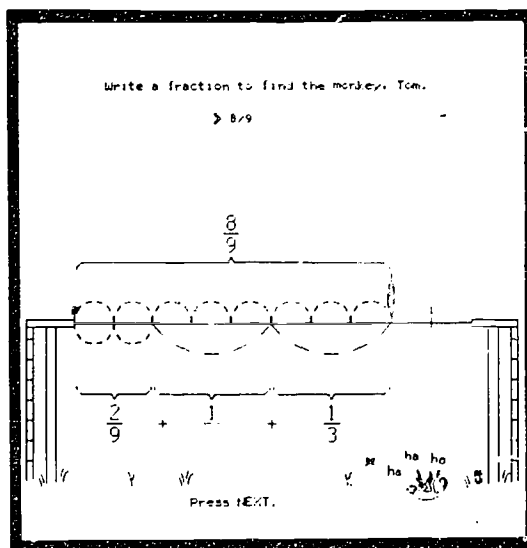
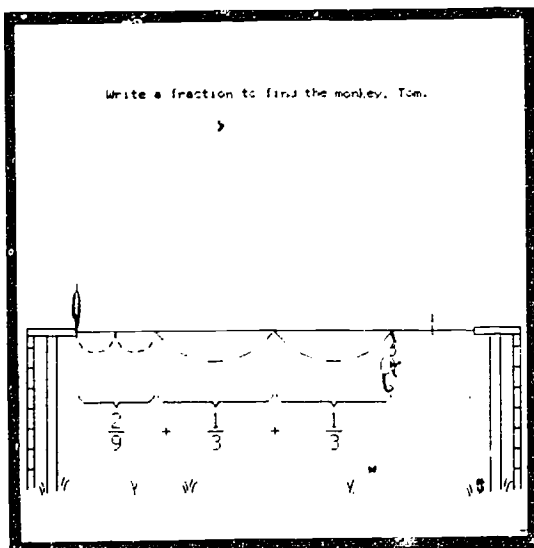
Purpose: Encourage the student to make up his or her own fraction addition problems and try painting problems in different ways.

Description: The student has a choice of three activities:

- 1) Write an addition number sentence for paintings done by PLATO (as in "Paint Addition: PLATO Paints")
- 2) Paint problems that PLATO chooses (as in "Paint Addition: You Paint")
- 3) Paint a box, then write an addition number sentence to describe the painting.

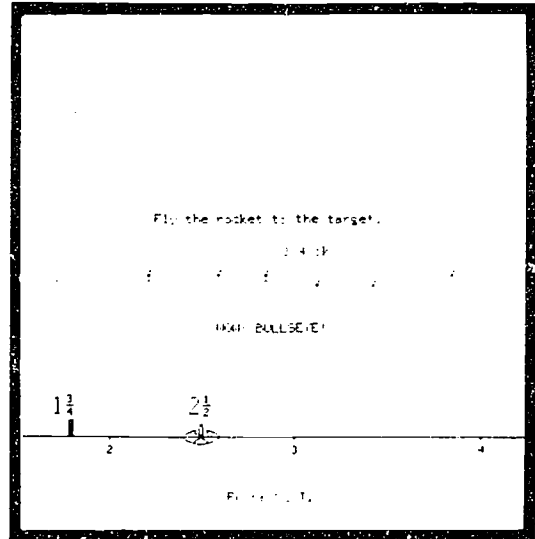
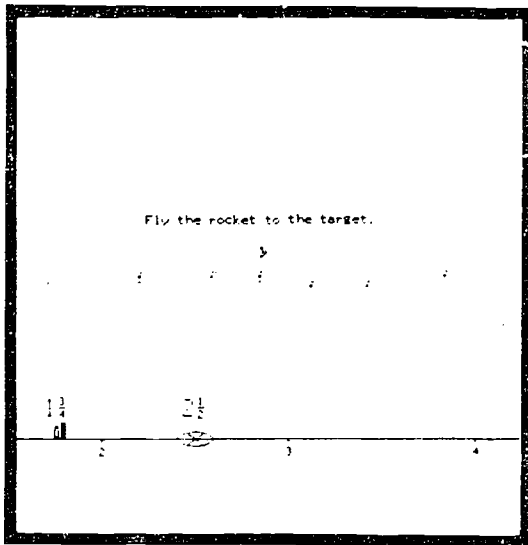
The student may save his or her painting in a library for others to see.

High Wire



Purpose: Show addition of fractions on the number line with visual feedback about common denominators.

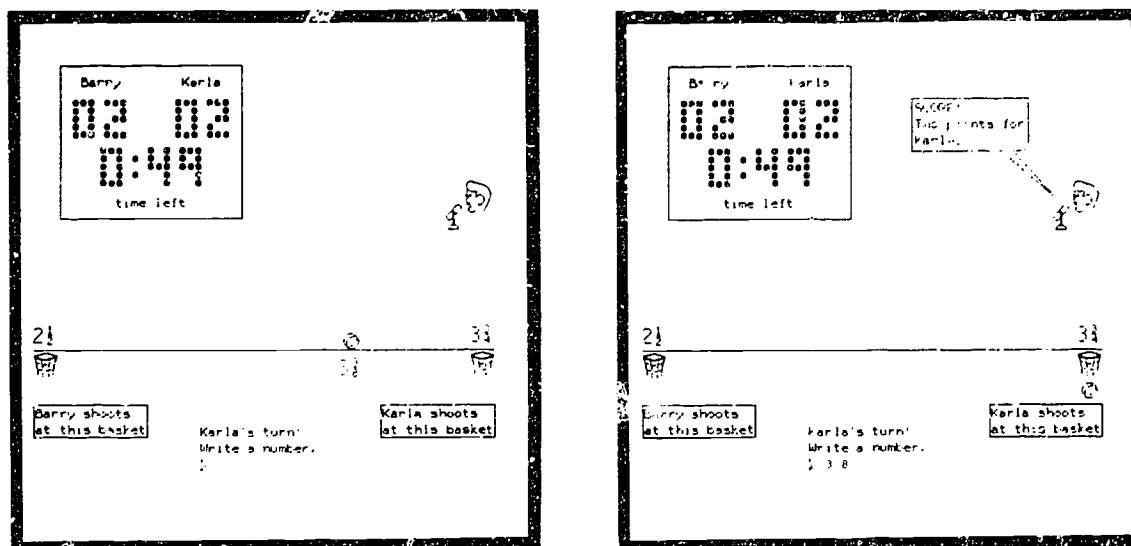
Description: A monkey swings along under a high wire (number line) making an addition problem (e.g., $\frac{2}{3} + \frac{1}{2}$ is 2 swings of length $\frac{1}{3}$ and 1 swing of length $\frac{1}{2}$). The student writes a fraction to hop a feather along above the number line. If the student is right, the feather lands where it can tickle the monkey, and the monkey falls off the line laughing. This is a number line illustration of common denominator; the monkey's swings and feather's hops match up to show equivalences. Difficulty adjusts to the student's performance.

Target Practice

Purpose: Addition practice which may encourage checking the answer by estimation.

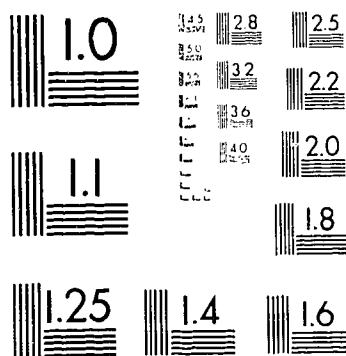
Description: The student fires a remote control rocket at a target. Both the rocket and the target are on a number line at known positions. An easy estimate will hit the target for a "SCORE," but only the exact answer gets a "BULLSEYE." Difficulty adjusts to the student's performance.

Fractions Basketball

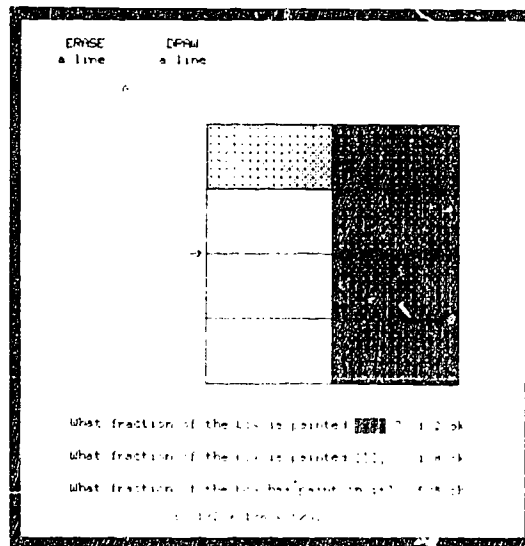


Purpose: Subtraction practice on the number line.

Description: This is a game for one or two students. Given ball and basket positions on the number line, the student bounces the ball some fraction or mixed number (e.g., $2\frac{7}{8}$ makes the ball do two bounces of length 1 then seven bounces of length $\frac{1}{8}$) to make a basket. Difficulty adjusts to the student's performance.



MICROCOPY RESOLUTION TEST CHART
 NATIONAL BUREAU OF STANDARDS
 STANDARD REFERENCE MATERIAL 1010a
 (ANSI and ISO TEST CHART No. 2)

Checkup: Paint Addition

There is a short checkup sequence available in lesson "Paint Addition: PLATO Paints." This sequence serves to review a visual approach to adding fractions and see if the student is ready for more advanced material.

ADDITION AND SUBTRACTION WITH UNLIKE DENOMINATORS --

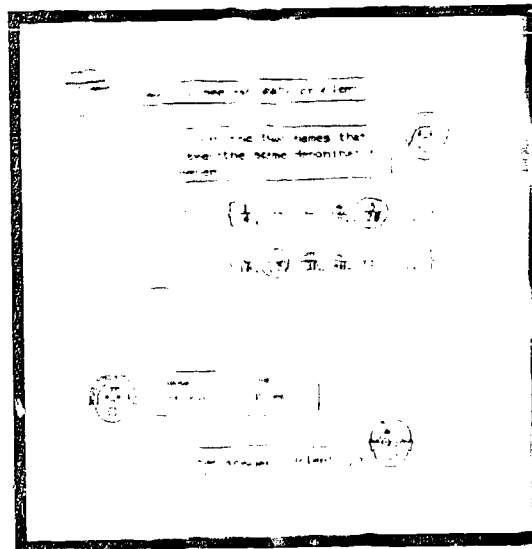
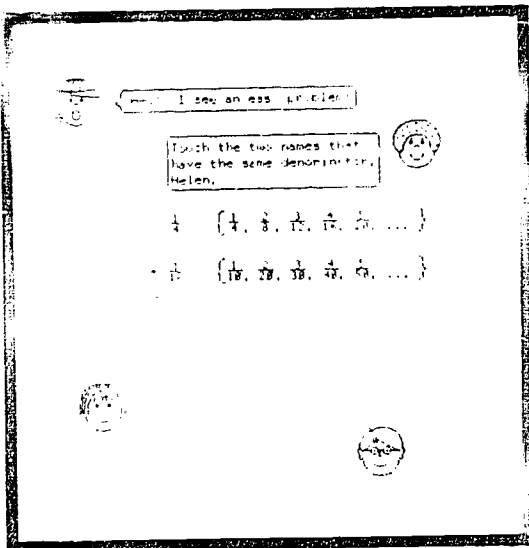
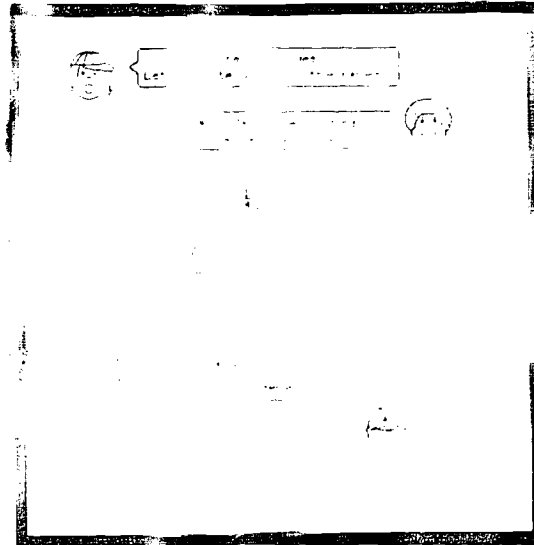
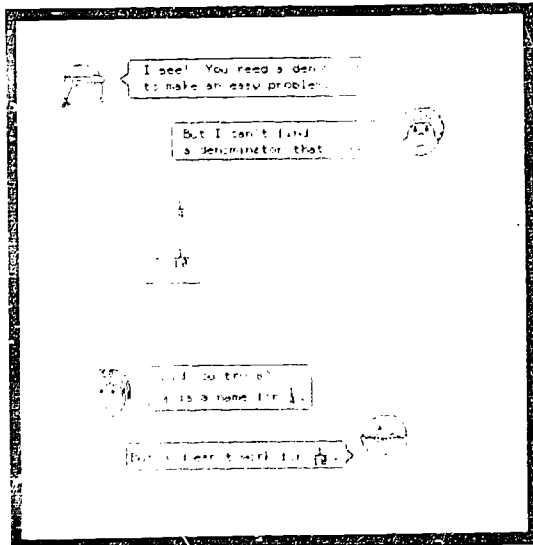
A SET LISTING APPROACH

This set of lessons develops a systematic method for finding common denominators in order to add and subtract mixed numbers. It is assumed that the student is familiar with the heuristic approach developed in the "Addition by Using an Easier Problem" series of lessons and is able to list an equivalence set for any fraction. The student learns to find a common denominator by listing equivalence sets for each of the given fractions. It is illustrated that more common denominators can be found by listing more fractions in the equivalence sets. There is some discussion of how to decide when enough fractions have been listed.

Addition with Equivalence Sets	1
Addition and Subtraction Practice -- Equivalence Sets	7
Methods for Finding Equivalence Sets	13
Checkup: Addition and Subtraction Unlike Denominators	17

Note: Lessons that require the use of a touch panel are identified by the word "touch" after the title on the lesson description page.

Addition With Equivalence Sets (touch)



Purpose: Suggest a systematic method for finding a common denominator.

Description: Given the problem $\frac{1}{4} + \frac{1}{10}$, the kids on the screen try to find an appropriate common denominator. After trying a few denominators that don't work, they decide to list the equivalence sets for $\frac{1}{4}$ and $\frac{1}{10}$. The student lists the sets, finds 2 names with a common denominator, and answers the problem.

Addition and Subtraction Practice With Equivalence Sets (touch)

Try this one.

List the equivalence set for $\frac{1}{4}$.

$$\begin{array}{r} 2\frac{1}{4} \quad \{ \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \dots \} \\ + 1\frac{1}{4} \quad \{ \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \dots \} \\ \hline \end{array}$$

Try this one.

$$\begin{array}{r} 2\frac{1}{4} \quad \{ \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \dots \} \\ + 1\frac{1}{4} \quad \{ \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \dots \} \\ \hline \end{array}$$

Touch two fractions that make an even whole.

Try this one.

$$\begin{array}{r} 2\frac{1}{4} \quad \{ \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \dots \} \\ + 1\frac{1}{4} \quad \{ \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \dots \} \\ \hline \end{array}$$

2 = no

Did you forget the whole numbers?

Try this one.

$$\begin{array}{r} 2\frac{1}{4} \quad \{ \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \dots \} \\ + 1\frac{1}{4} \quad \{ \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \dots \} \\ \hline \end{array}$$

3 = 1

Right number, Helen.
Now write it in standard form.

Purpose: Provide experience adding and subtracting fractions and mixed numbers, using a systematic method to find a common denominator.

Description: The student solves addition and subtraction problems by listing 5 elements in the equivalence set for each fraction, then picking out 2 names with the same denominator. Difficulty adjusts to the student's performance. This lesson involves no strategies for deciding how many names should be listed for each fraction. A student who does well can finish the lesson in as few as 3 problems and be ready for the lesson, "Strategies for Listing Equivalence Sets."

Strategies for Listing Equivalence Sets (touch)

Hey! I see another set problem!

So to 1!

1/2 + 1/3

1/2 { 1/2, 2/4, 3/6, 4/8, 5/10, ... }

1/3 { 1/3, 2/6, 3/9, 4/12, 5/15, ... }

3/6 + 2/6 = 5/6

That makes 3/6 + 2/6 = 5/6

So we don't need any more names.

1/2 { 1/2, 2/4, 3/6, 4/8, 5/10, ... }

1/3 { 1/3, 2/6, 3/9, 4/12, 5/15, ... }

Write as many names as you want.

Press Enter to move to the other set.

LRB when solving for the missing names.

- Purpose:
- 1) Provide practice adding and subtracting fractions and mixed numbers, using equivalence sets to find a common denominator.
 - 2) Point out that more common denominators can be found by listing more names.
 - 3) Develop strategies for deciding how many names to list to find a common denominator.
 - 4) Enable the student to solve addition and subtraction problems without relying on guidance from PLATO.

Description: There are 2 introductory problems followed by practice. When the student finishes the first problem, kids on the screen point out that there are 2 common denominators visible in the set listings. They see that either one can be used to find the correct answer to the problem.

They then list more names to find another common denominator and speculate about how many more common denominators could be found.

The student starts to solve the second problem by listing the equivalence sets. As soon as a common denominator is found in the second set, the kids on the screen point out that there is no need to list more names. In the practice problems that follow, the student is free to list as many names for each fraction as he or she wants. Before writing the answer, the student must touch 2 names that have like denominators. For students who have trouble listing the sets, PLATO sometimes offers to calculate the equivalent fractions or let the student can view the overall addition process without getting bogged down in the individual calculations. Difficulty adjusts to the student's performance. For higher levels of difficulty, the student is not required to list sets or touch 2 names that have like denominators. The student can do whatever set-listing he or she needs.

Checkup: Addition and Subtraction with Unlik. Denominators

$$\begin{array}{r} 3\frac{1}{2} \\ - 1\frac{1}{3} \\ \hline \end{array}$$

Write the answer.

Or press to list the equivalence sets.

$$\begin{array}{r} 3\frac{1}{2} \\ - 1\frac{1}{3} \\ \hline \end{array}$$

Write the answer.

Press to list the equivalence sets.

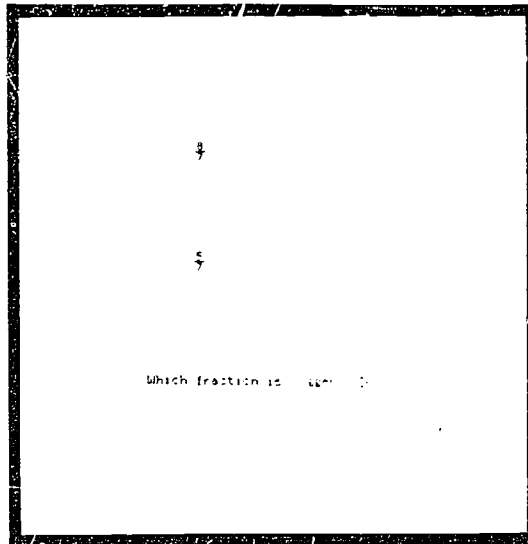
There is a short checkup sequence available in lesson "Strategies for Listing Equivalence Sets." The student is asked to solve addition and subtraction problems with unlike denominators. The student has the option of listing equivalent sets.

INEQUALITIES

This set of lessons develops a systematic method for comparing the size of two fractions. Two fractions are compared by listing an equivalence set for each, then comparing names that have the same denominator. It is assumed that the student is already able to list an equivalence set for any fraction.

Inequalities with Like Denominators	M3
Inequalities with Unlike Denominators	M4
Checkup: Inequalities	M5

Note: Lessons that require the use of a touch panel are identified by the word "touch" after the title on the lesson description page.

Inequalities With Like Denominators

Purpose: Provide readiness for comparing fractions with unlike denominators.

Description: The student tells which of 2 fractions with like denominators is bigger. If the student is not correct on the first try, PLATO displays the appropriate fractions of pizza for comparison.

Inequalities With Unlike Denominators (touch)

$\frac{1}{3}$
 $\frac{1}{4}$

Which fraction is bigger? >
 If they are the same, write "same."

If you want to list the equivalence sets, press HELP.

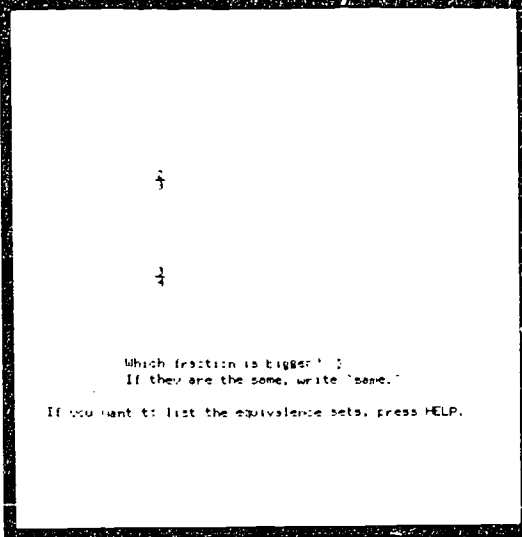
$\frac{1}{3}$ { $\frac{1}{3}, \frac{2}{6}, \frac{4}{12}, \dots$ }
 $\frac{1}{4}$

Write as many names as you want.

Press DATA to move to the other set.
 LFB when you are done listing names.

Purpose: Develop a systematic method for comparing the size of 2 fractions with unlike denominators.

Description: Given 2 fractions with unlike denominators, the student tells which is bigger or that they're equivalent. Before responding, the student can choose to press HELP for a chance to list some names for either or both of the fractions. If the student's response is incorrect, he or she must list both equivalence sets and touch 2 names that have a common denominator before trying the question again.

Checkup: Inequalities

$\frac{2}{3}$

$\frac{1}{4}$

Which fraction is bigger? :
If they are the same, write "same."

If you want to list the equivalence sets, press HELP.

There is a short checkup sequence available in lesson "Inequalities with Unlike Denominators" to see if the student has mastered the topic.

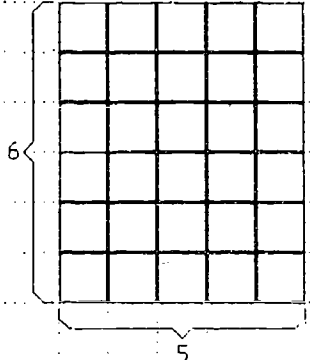
MULTIPLICATION OF MIXED NUMBERS

This set of lessons covers multiplication of mixed numbers and fractions. The basic model used is an array of squares (like area). For example, $2 \times 3 \frac{1}{2}$ is illustrated by 2 rows of $3 \frac{1}{2}$ squares each. It is assumed that the student is familiar with the array model of multiplication for whole numbers. After this model is extended to illustrate multiplication of fractions, a "short cut" $[a/b \times c/d = (axc)/(bxd)]$ is discussed. Various methods, including the array, repeated addition, and the short cut, are discussed. The student is encouraged to examine each problem and choose an appropriate method for the given type of problem.

Introduction to Multiplication of Mixed Numbers	N1
Multiplication Practice with Pictures for Help	N4
Multiplication of Fractions	N5
Strategies for Multiplying Mixed Numbers	N6
Mixed Number Multiplication Practice	N7
Checkup: Multiplication of Mixed Numbers	N8

Note: Lessons that require the use of a touch panel are identified by the word "touch" after the title on the lesson description page.

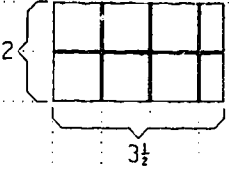
Introduction to Multiplication of Mixed Numbers



Write a multiplication number sentence about the boxes.

$$6 \times 5 = 30$$

Great, Marilyn!

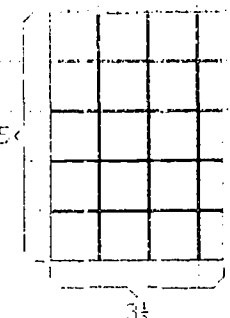


Let's put another half box in each row!

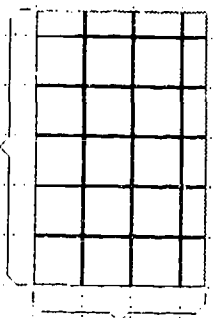
Now there are $3\frac{1}{2}$ boxes in each row!

For help, press HELP.

How many boxes are there all together, Marilyn?



How many boxes all together? $17\frac{1}{2}$!

$$5 \times 3\frac{1}{2} = 17\frac{1}{2}$$


Write a multiplication number sentence about the boxes.

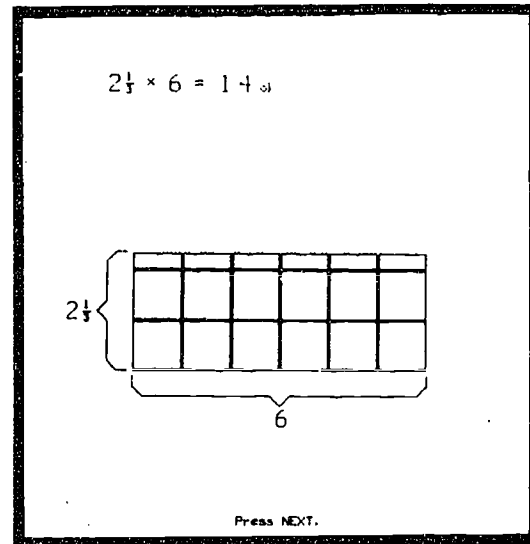
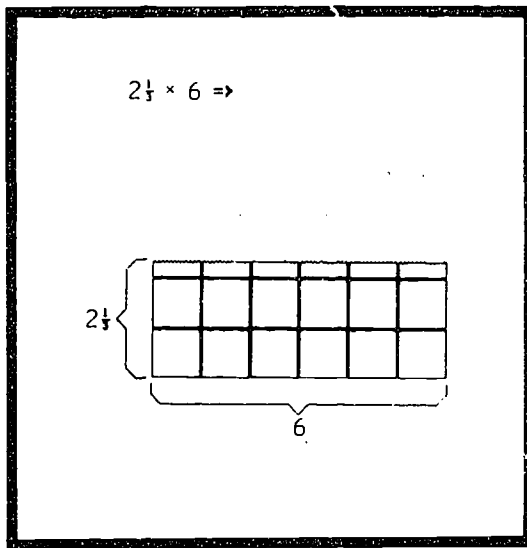
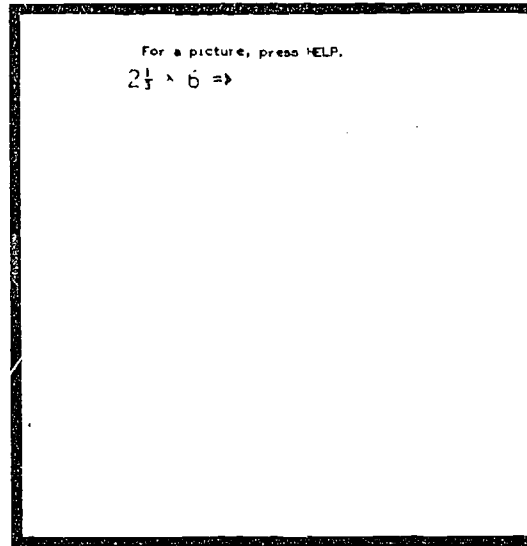
$$5\frac{1}{2} \times 3\frac{1}{2} = 19\frac{1}{4}$$

Great, Marilyn!

Purpose: Introduce multiplication of mixed numbers, using a familiar multiplication model.

Description: The student writes multiplication number sentences to describe arrays of boxes. After the student shows proficiency with problems involving only whole numbers, an extra half box is added to each row. The student is asked to tell how many boxes there are by thinking of putting the half boxes together to make whole boxes. Later problems ask the student to write multiplication number sentences for arrays involving halves and thirds of boxes. Difficulty adjusts to the student's performance.

Multiplication Practice with Pictures for Help




Purpose: Practice multiplication of mixed numbers, with a familiar model available as the student needs it.

Description: PLATO presents mixed number multiplication problems one at a time. The fractions involved are limited to $\frac{1}{2}$ and $\frac{1}{3}$ so that the problems can be easily visualized as arrays of boxes. The student can press the HELP key any time to get a picture representing the problem. Difficulty adjusts to the student's performance.

Multiplication of Fractions

You have learned to multiply mixed numbers by thinking of rows and boxes.

Here is a problem with just fractions.

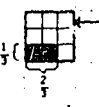
$$\frac{1}{2} \times \frac{1}{2}$$


Think of $\frac{1}{2}$ of a row with $\frac{1}{2}$ of a box in each row.

What fraction of a box does that make? $\frac{1}{4}$ Right!

So $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

Try this one.

$$\frac{1}{3} \times \frac{2}{3}$$


A whole box would be this big.

And here are some lines on the box to help you.

Think of $\frac{1}{3}$ of a row with $\frac{2}{3}$ of a box in each row.

What fraction of a box does that make?

PROBLEMS YOU HAVE DONE

$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$	$\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$
$\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$	$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

Look at the problems Helen has done!

You can multiply the top parts to get the top part of the answer.

I see! And you can multiply the bottom parts to get the bottom part of the answer!

$1 \times 1 = 1$ ok.

$\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

$2 \times 2 = 4$ ok.

It works for this one! NEXT for another.

Let's make sure the short-cut works for all of them.

PROBLEMS YOU HAVE DONE

$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ ✓	$\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$	$\frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$
$\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$	$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$	$\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$

Try the multiplication short-cut.

$\frac{1}{8} \times \frac{1}{8} = \frac{1}{64}$ Right!

$\frac{1}{6} \times \frac{2}{3} = \frac{2}{18}$ Right!

$\frac{1}{4} \times \frac{2}{3} = \frac{2}{12}$

To see the short-cut, press HELP.

- Purpose:**
- 1) Begin multiplication of fractions, using a familiar multiplication model.
 - 2) Introduce an algorithm for multiplication of fractions.

Description: The student does six multiplication problems using arrays of boxes as in previous lessons. All of the problems involve pure fractions rather than mixed numbers, so the student must think of a fraction of a row with a fraction of a box in each row. The finished problems are saved at the bottom of the screen. After the six problems are done, some kids on the screen notice a pattern in the problems and suggest a short-cut for finding the answers. With the student's help they verify that the short-cut works for each problem. PLATO then raises the question of why the short-cut works and offers the student the option of seeing an explanation. Practice problems follow. Difficulty adjusts to the student's performance.

Strategies for Multiplying Mixed Numbers

Here is a problem.

$$2\frac{1}{2} \times 6$$

Here is a way to find the answer.

Think of $2\frac{1}{2}$ rows with 6 boxes in each row.

How many boxes all together? 15 ok

So $2\frac{1}{2} \times 6 = 15$

Here is a problem.

$$2\frac{1}{2} \times 6$$

Here is a way to find the answer.

Think of $2\frac{1}{2}$ rows with 6 boxes in each row.

How many boxes all together? 15 ok

So $2\frac{1}{2} \times 6 = 15$ Right!

That's a good way. But this way is easier for me.

I just think of $2\frac{1}{2}$ sixes.

2 sixes are 12 ok

$\frac{1}{2}$ of a six is 3 ok

So $2\frac{1}{2}$ sixes are 15 ok

So $2\frac{1}{2} \times 6 = 15$

Think of different ways to find the answer.
Choose a way that is easy for you.

$$2\frac{1}{2} \times 3 = ?$$

You could think of $2\frac{1}{2}$ threes.

HELP for another hint.

Think of different ways to find the answer.
Choose a way that is easy for you.

$$2\frac{1}{2} \times 3 = 7 \text{ Right!}$$

$$3\frac{1}{2} \times 2 = 7 \text{ Right!}$$

$$4\frac{1}{2} \times 4 = 18 \text{ Right!}$$

$$1\frac{3}{4} \times 5 = ?$$

HELP for a hint.

- Purpose:
- 1) Demonstrate that there are many ways to multiply mixed numbers.
 - 2) Encourage the student to examine each problem and choose a method appropriate for the particular problem.

Description: PLATO presents a multiplication problem, and kids on the screen discuss different ways it could be solved. The methods suggested involve concepts such as multiplying by repeated addition or making an array of boxes. The student then works problems similar to the one discussed. When that type of problem is mastered, a new type is discussed and practiced. Each practice set reinforces previous types of problems as well as working on the type being emphasized. On any problem the student can press the HELP key for a "hint." The hint suggests a method appropriate for that problem. Two hints are available for each problem.

Mixed Number Multiplication Practice

For a hint, press HELP.

$$3 \times 4\frac{1}{2} = 13\frac{1}{2} \text{ ok} \quad \frac{1}{2} \times \frac{1}{4} = \frac{1}{8} \text{ ok}$$

$$5 \times \frac{1}{4} = 1\frac{1}{4} \text{ ok} \quad 1\frac{1}{4} \times 2 = 3\frac{1}{2}$$

For a hint, press HELP.

$$3 \times 4\frac{1}{2} = 13\frac{1}{2} \text{ ok} \quad \frac{1}{2} \times \frac{1}{4} = \frac{1}{8} \text{ ok}$$

$$5 \times \frac{1}{4} = 1\frac{1}{4} \text{ ok} \quad 1\frac{1}{4} \times 2 = 3\frac{1}{2}$$

You could think of fraction names for $1\frac{1}{4}$ and 2, then use the short-cut.

Purpose: Practice multiplying mixed numbers.

Description: The student does mixed number multiplication problems. On any problem the student can press the HELP key for a "hint." The hint suggests one method appropriate for the particular problem. Later in the lesson the student is asked to do similar problems with no hints available.

Checkup: Multiplication of Mixed Numbers

This is a CHECKUP. Do your best!

$5 \times 3\frac{1}{2} = 16$ ok	$\frac{1}{3} \times 2\frac{1}{2} = \frac{5}{6}$ ok
$5 \times \frac{1}{4} = 1\frac{1}{4}$ ok	$1\frac{1}{2} \times 2\frac{1}{2} = 3\frac{3}{4}$ ok
$\frac{1}{3} \times 3 = 1$ ok	$5 \times 4\frac{2}{3} = 22$ ok
$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ ok	$\frac{1}{8} \times \frac{3}{4} = \frac{3}{32}$ ok
$1\frac{1}{2} \times 4 = 5\frac{1}{2}$ ok	$4\frac{1}{2} \times 2 = 9$ no
$2 \times 3\frac{1}{2} = 7$ ok	Try once more
$2\frac{1}{4} \times 4 = 9$ ok	
$\frac{1}{3} \times 6 = 2$ ok	

This is a CHECKUP. Do your best!

$5 \times 3\frac{1}{2} = 16$ ok	$\frac{1}{3} \times 2\frac{1}{2} = \frac{5}{6}$ ok
$5 \times \frac{1}{4} = 1\frac{1}{4}$ ok	$1\frac{1}{2} \times 2\frac{1}{2} = 3\frac{3}{4}$ ok
$\frac{1}{3} \times 3 = 1$ ok	$5 \times 4\frac{2}{3} = 22$ ok
$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ ok	$\frac{1}{8} \times \frac{3}{4} = \frac{3}{32}$ ok
$1\frac{1}{2} \times 4 = 5\frac{1}{2}$ ok	$4\frac{1}{2} \times 2 = 9$ no
$2 \times 3\frac{1}{2} = 7$ ok	$\frac{1}{3} \times 4 = 1\frac{1}{3}$ ok
$2\frac{1}{4} \times 4 = 9$ ok	$3\frac{1}{4} \times \frac{1}{2} = 1\frac{3}{8}$ ok
$\frac{1}{3} \times 6 = 2$ ok	

- Purpose:**
- 1) Review multiplication of mixed numbers.
 - 2) See if the student is ready for material which depends on multiplication of mixed numbers.

Description: The check-up consists of fifteen multiplication problems, each of a different basic type. The student is allowed two tries on each problem.

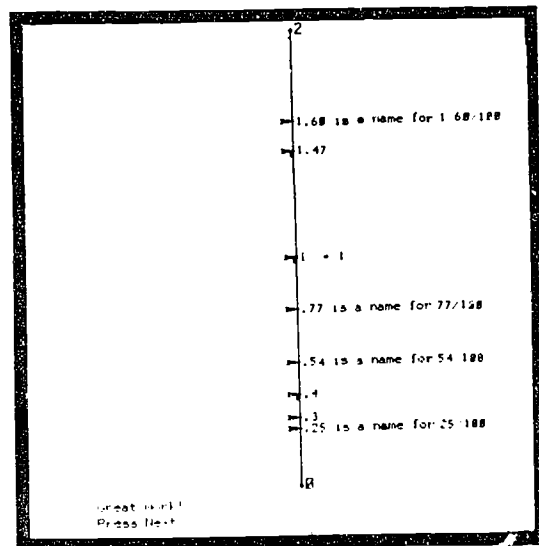
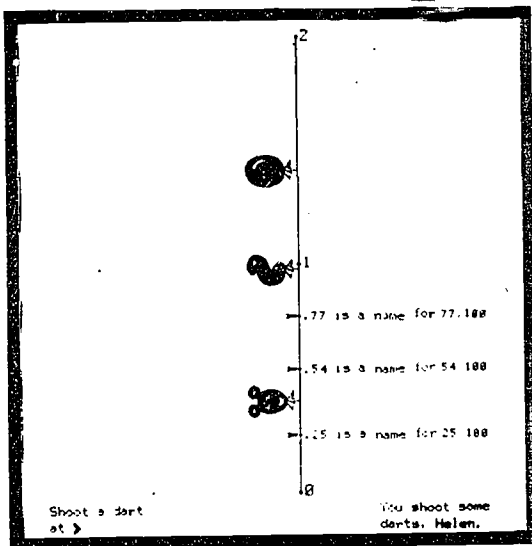
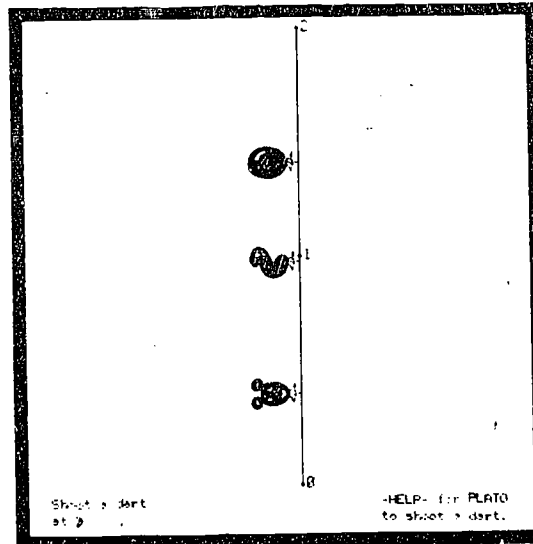
MEANING OF DECIMAL FRACTIONS

This set of lessons uses the number line to introduce decimal fractions. Although the initial introduction relates decimal fractions to common fractions, decimal numbers are treated primarily as a self-contained system. The lessons emphasize placement and order of decimal numbers on the number line. Estimating distances with decimal numbers is also done.

Decimal Darts, Part 1	03
Decimal Darts, Part 2	04
Decimal Torpedo	05
Obstacle Course	06
Checkup: Decimal Darts	07

Note: Lessons that require the use of a touch panel are identified by the word "touch" after the title on the lesson description page.

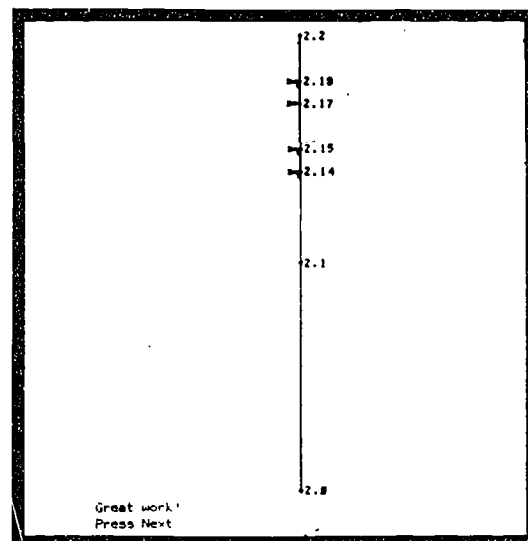
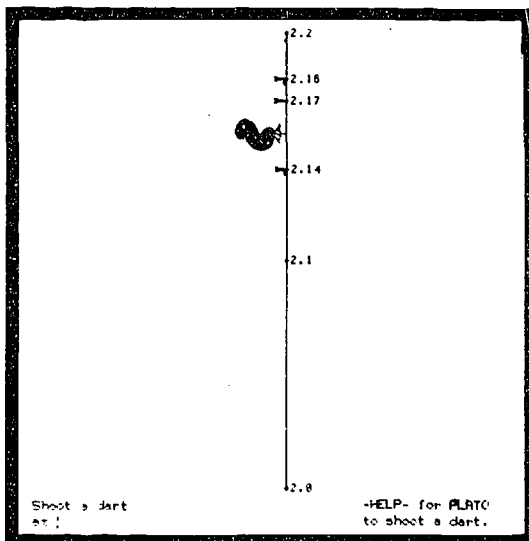
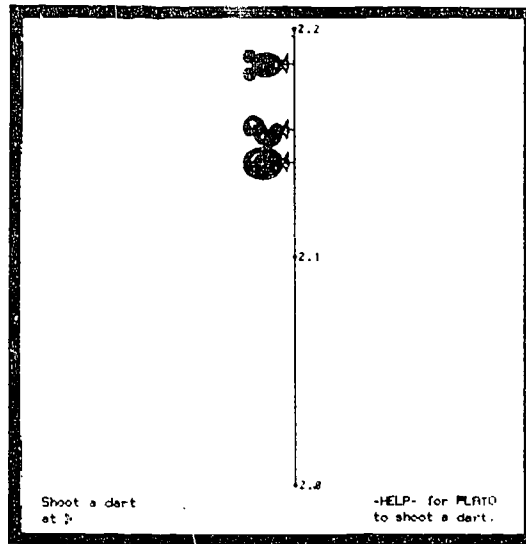
Decimal Darts, Part 1



Purpose: Introduce decimal names for tenths and hundredths and provide experience using these decimal names to locate points on the number line.

Description: After a short introduction to decimal names, the student writes numbers to shoot darts at balloons tied to a number line. Common fraction names and division are not accepted, so the student must use decimal names. The student can press HELP to have PLATO shoot a dart. Difficulty adjusts to the student's performance.

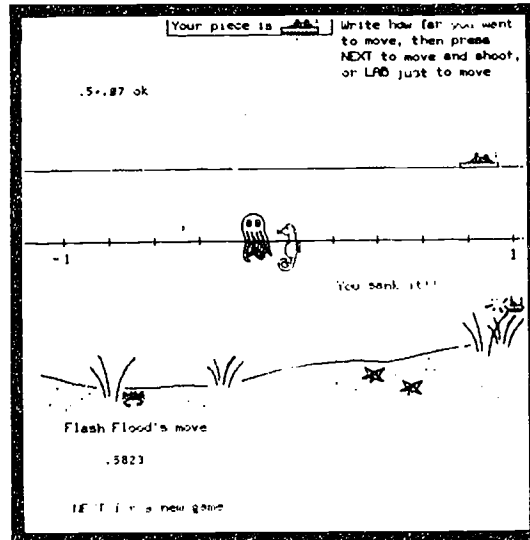
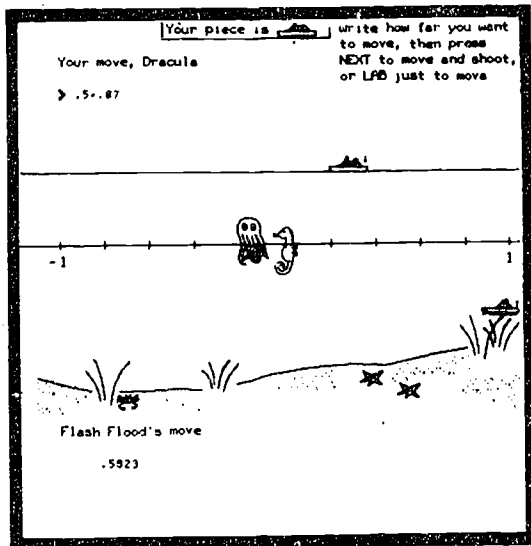
Decimal Darts, Part 2



Purpose: Introduce decimal names for thousandths and provide further experience using decimal names to locate points on the number line.

Description: This lesson is like "Decimal Darts, Part 1" except that the endpoints of the number line are usually tenths rather than integers. It is necessary to use thousandths to hit some of the balloons. Difficulty adjusts to the student's performance. After showing proficiency in using decimal names, the student is permitted to use both decimal and common fraction names.

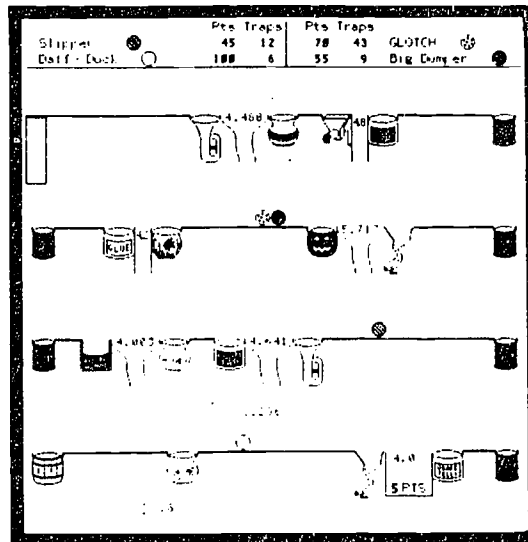
Decimal Torpedo



Purpose: Practice using decimal numbers to estimate fractional distances on a number line.

Description: This is an iterterminal number line game. Two students compete. One controls a boat, the other a submarine. The object is to move to a point above (or below) the apponent's piece and shoot it. Common fraction names and division are not accepted, so the student must use decimal names and expressions to move fractional distances. The number line varies from game to game.

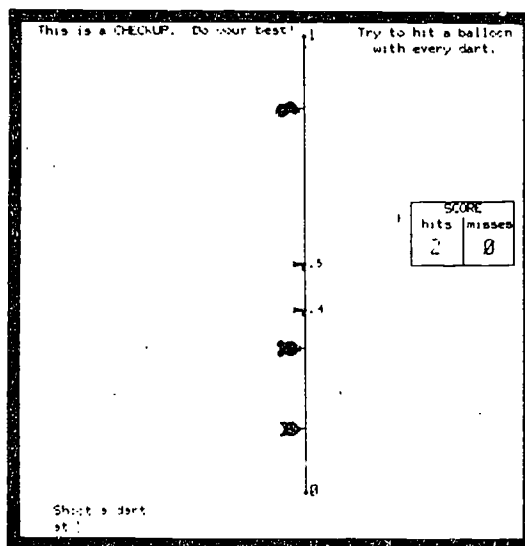
Obstacle Course



Purpose: Provide experience with addition, subtraction, and estimation with decimals.

Description: This is an interterminal game. The student uses decimal numbers to move a ball along a number line, trying to maneuver through the chutes and avoid the various "traps." The student can choose the level of difficulty. Up to four students can play on the same "course" at the same time.

Checkup: Decimal Darts



Purpose: Review placement of decimal numbers on the number line and see if the student has mastered the topic.

Description: The student writes decimal numbers to shoot darts at balloons tied to a number line. PLATO displays the student's score of "hits" and "misses."

CONVERSIONS BETWEEN COMMON FRACTIONS AND DECIMAL FRACTIONS

This set of lessons relates decimal fractions to common fractions. Although there is no instruction presented, there is help available as the student encounters new types of problems. The fractions presented have denominators of 10, 100, or 1000. These lessons stress converting common fractions and mixed numbers to decimal numbers, and vice versa. There is also some work with identifying sets of names for the same number, for example, $1\frac{2}{10}$, 1.2, 1.20, $1\frac{20}{100}$, and $1\frac{1}{5}$.

Decimal Names for Fractions	P3
Fraction Names for Decimal Numbers	P4
Sort Number Names with Decimals	P5
Decimal Pinball	P6
Checkup: Decimal and Fraction Conversions . . .	P7

Note: Lessons that require the use of a touch panel are identified by the word "touch" after the title on the lesson description page.

Decimal Names for Fractions

Write a decimal name for the given number.
If you need help, press HELP.

$\frac{2}{10} = .2$ ok

$\frac{8}{10} = .8$ ok

$\frac{3}{10} = .3$ ok

$2\frac{9}{10} = 2.9$ ok

$\frac{7}{10} = .7$ ok

$12\frac{4}{10} = 12.4$ ok

$18\frac{5}{10} = 18.5$ ok

-HELP- available

Write a decimal name for the given number.
If you need help, press HELP.

$\frac{2}{10} = .2$ ok

$\frac{8}{10} = .8$ ok

$\frac{3}{10} = .3$ ok

$2\frac{9}{10} = 2.9$ ok

$\frac{7}{10} = .7$ ok

$12\frac{4}{10} = 12.4$ ok

$18\frac{5}{10} = 18.5$ ok

$\frac{1}{10} = .1$ ok

$10\frac{6}{10} = 10.6$ ok

$\frac{7}{10} = .7$ ok

$\frac{96}{100} = .96$ ok

$\frac{97}{100} = .97$ ok

$\frac{4}{10} = .4$ ok

$\frac{46}{100} = .46$ ok

$\frac{1}{10} = .1$ ok

$\frac{45}{100} = .45$ ok

$\frac{9}{10} = .9$ ok

$\frac{83}{100} = .83$ ok

Press NEXT to do more of these.
SHIFT-NEXT to do something else.

Purpose: Practice writing decimal names for tenths, hundredths, and thousandths.

Description: The student is asked to write decimal names for the common fractions and mixed numbers given by PLATO. Difficulty adjusts to the student's performance.

Fraction Names for Decimal Numbers

Write a fraction or mixed number name for the given number.
If you need help, press HELP.

$8.3 = \frac{3}{10}$ ok	$8.88 = \frac{88}{100}$ ok
$8.8 = \frac{8}{10}$ ok	$15.1 = 15\frac{1}{10}$ ok
$8.9 = \frac{9}{10}$ ok	$8.2 = \frac{2}{10}$ ok
$12.4 = 12\frac{4}{10}$ ok	$8.3 = \frac{3}{10}$ ok
$8.7 = \frac{7}{10}$ ok	$8.52 \rightarrow$
$2.1 = 2\frac{1}{10}$ ok	
$8.2 = \frac{2}{10}$ ok	
$2.5 = 2\frac{5}{10}$ ok	
$8.6 = \frac{6}{10}$ ok	

Write a fraction or mixed number name for the given number.
If you need help, press HELP.

$8.89 = \frac{89}{100}$ ok	$8.88 = \frac{88}{100}$ ok
$8.45 = \frac{45}{100}$ ok	$12.73 = 12\frac{73}{100}$ ok
$8.87 = \frac{87}{100}$ ok	$18.2 = 18\frac{2}{10}$ ok
$8.5 = 8\frac{5}{10}$ ok	$12.86 = 12\frac{86}{100}$ ok
$8.84 = \frac{84}{100}$ ok	$18.81 = 18\frac{81}{100}$ ok
$8.83 = \frac{83}{100}$ ok	$28.63 = 28\frac{63}{100}$ ok
$1.97 = 1\frac{97}{100}$ ok	$9.88 = 9\frac{88}{100}$ ok
$4.3 = 4\frac{3}{10}$ ok	$7.85 = 7\frac{85}{100}$ ok
$8.41 = 8\frac{41}{100}$ ok	$8.82 = \frac{82}{100}$ ok

Press NEXT to do more of these.
SHIFT-NEXT to do something else.

Purpose: Practice writing common fraction and mixed number names for numbers expressed as decimals.

Description: The student is asked to write fraction or mixed number names for tenths, hundredths, and thousandths written by PLATO as decimals. Difficulty adjusts to the student's performance. On the highest difficulty level some of the problems give a fraction name and ask the student to write a decimal name.

Sort Number Names with Decimals (touch)

Names for .47

Names for 4.7

$\frac{47}{100}$ 8.47 $\frac{7}{10}$

$\frac{128}{100}$ 4.70 $\frac{114}{100}$

$\frac{121}{100}$ $\frac{47}{10}$

Touch here to erase the ☐ .

Put the numbers inside the right loops.

When you're done, Press -LAP-

Names for .47

Names for 4.7

8.47 $\frac{47}{100}$

$\frac{128}{100}$ 4.70 $\frac{121}{100}$ $\frac{7}{10}$ $\frac{114}{100}$

$\frac{47}{10}$

Touch here to erase the ☐ .

1 number is not in the right loops.

When you're done, Press -LAP-

Names for .47

Names for 4.7

8.47 $\frac{47}{100}$

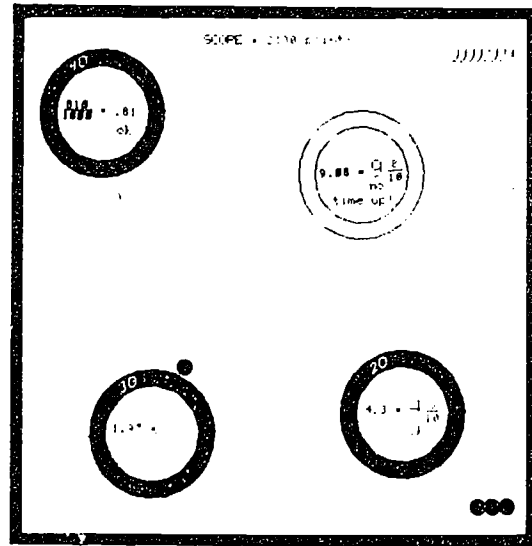
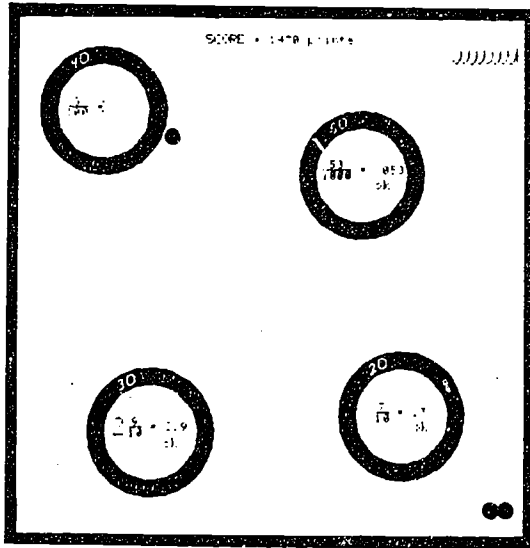
$\frac{128}{100}$ 4.70 $\frac{47}{10}$ $\frac{121}{100}$ $\frac{7}{10}$ $\frac{114}{100}$

Touch here to erase the ☐ .

Very good!
Press -NEXT-

Purpose: Practice identifying equivalent whole numbers, fractions, mixed numbers, and decimal numbers.

Description: Several numbers (whole numbers, fractions, mixed numbers, and decimal numbers) are scattered on the screen along with two, three, or four loops. The student sorts the numbers into the loops so that each loop contains an equivalence set. Difficulty adjusts to the student's performance.

Decimal Pinball

Purpose: Practice converting common fractions and mixed numbers to decimal numbers.

Description: The student writes decimal names for fractions and mixed numbers as a ball bounces from problem to problem as in a pinball machine. There is a time limit on each problem. The fractions used have denominators of 10, 100, or 1000. Points are earned for correct answers, and the student's score is displayed at the top of the screen. The current top twenty scores are saved and displayed with the students' names in a Decimal Pinball Hall of Fame.

Checkup: Decimal and Fraction Conversions

This is a CHECKUP. Do your best.

If PLATO gives a fraction or mixed number, you give a decimal name for the same number.

If PLATO gives a decimal name, you give a fraction or mixed number name for the same number.

$7\frac{8}{100} = 7.8$ ok $\frac{818}{1000} = .81$ ok

$8.81 = \frac{1}{100}$ ok $8.51 \div$

$19.848 = 19\frac{48}{1000}$ ok

$14.89 = 1 + \frac{9}{100}$ ok

$\frac{53}{1000} = .053$ ok

Purpose: Review conversion between common fractions and decimal fractions and see if student has mastered the topic.

Description: The student converts decimal numbers to common fractions and mixed numbers, and vice versa. Problems include tenths, hundredths, and thousandths.

Appendix I

Students' Sharing of Creative Work

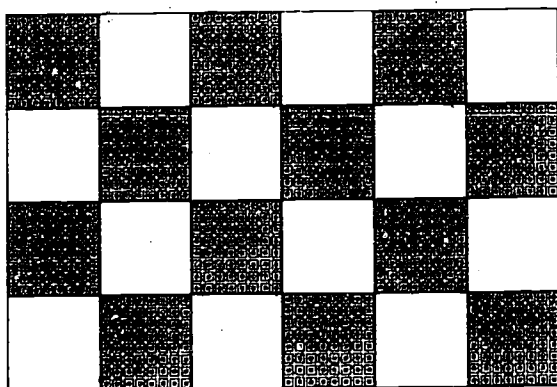
The student's work involves answering questions and working problems, but it also includes creative activities and sharing of ideas with other students. One way that this is accomplished is by means of "library" lessons. These are described at some length here because they are a prominent example of a way in which PLATO can provide important learning experiences that otherwise are difficult or impossible to provide.

In a library lesson a student does some mathematically relevant creative work and is then given the option of saving his work in a "library" so that other students can see it. He may replace his work with something new at any time. Each class has a separate library. Students are able to see the work stored in other classes' libraries, as well as their own. This provides an ongoing exchange of ideas among students in many different classrooms, even in different schools and communities.

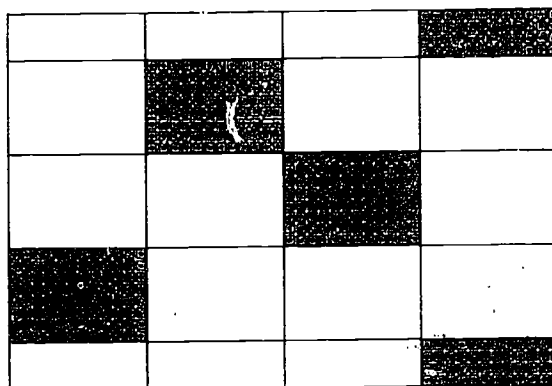
In the "Paintings Library" the student is asked to "paint" a particular fraction of a box. By touching the screen, he can draw lines on the box and paint areas of the box. When he has finished painting the correct fraction of the box, the student can choose to save his painting in the library. The following pages show a few such student paintings.

Social interaction is quite apparent when fads spread through the libraries. For example, one student might paint a fraction of the box to show his initials. Within a few days, there will be many paintings showing other students' names and initials.

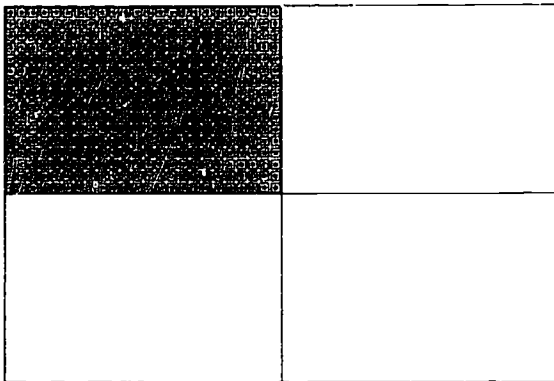
This is how Jennifer painted $1/2$ of the box.



This is how Beth painted $1/4$ of the box.



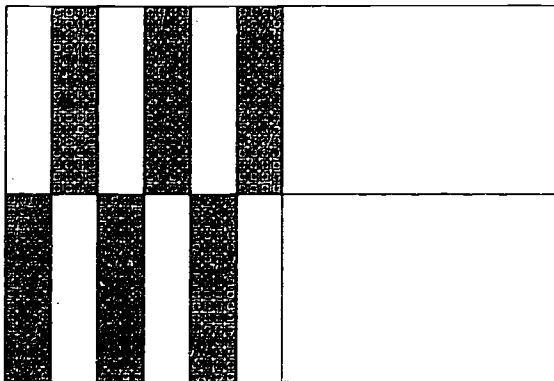
This is how martha painted $\frac{1}{4}$ of the box.



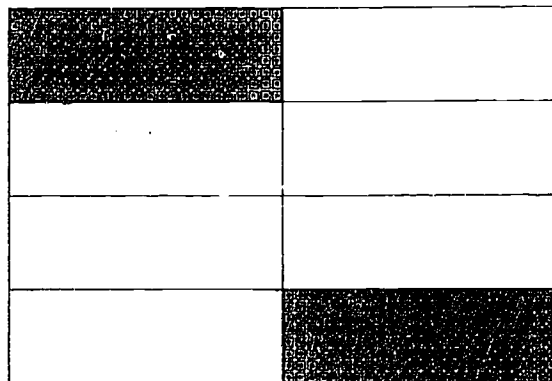
This is how vanaldo painted $\frac{1}{4}$ of the box.



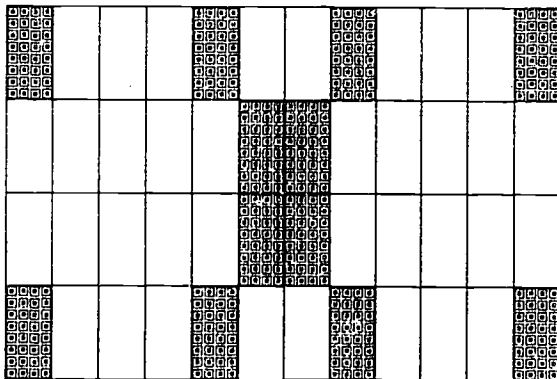
This is how linda painted $\frac{1}{4}$ of the box.



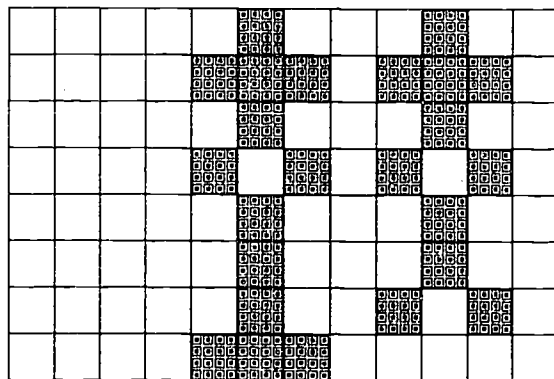
This is how julie painted $\frac{1}{4}$ of the box.



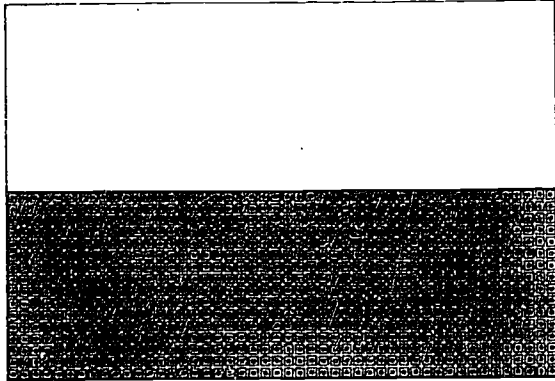
This is how alice painted $\frac{1}{4}$ of the box.



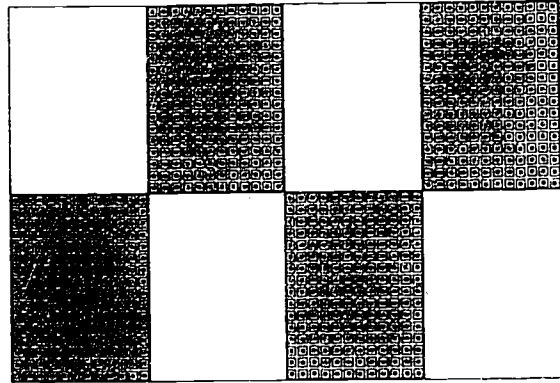
This is how dawn painted $\frac{1}{4}$ of the box.



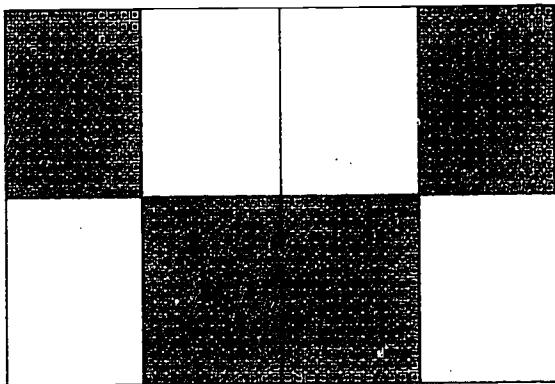
This is how lovetta h painted $1/2$ of the box.



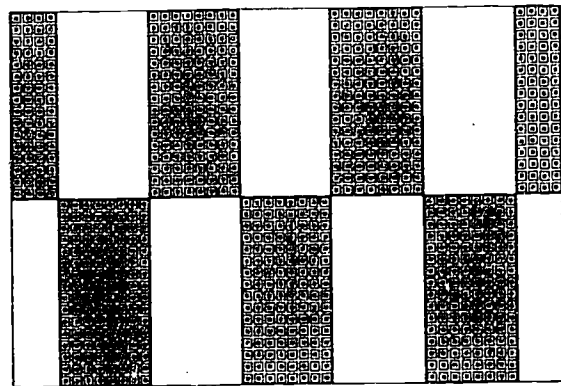
This is how craig j painted $1/2$ of the box.



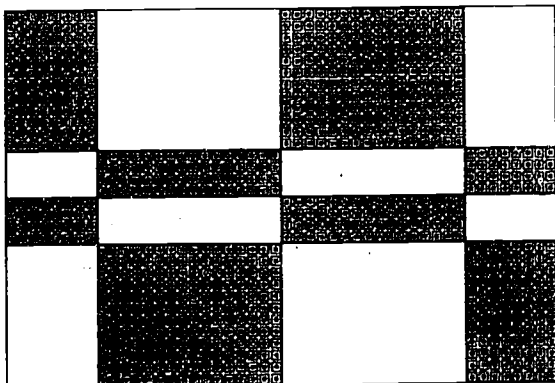
This is how steven s painted $1/2$ of the box.



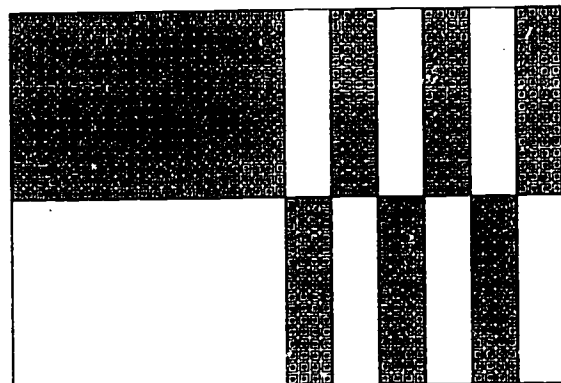
This is how anthia s painted $1/2$ of the box.



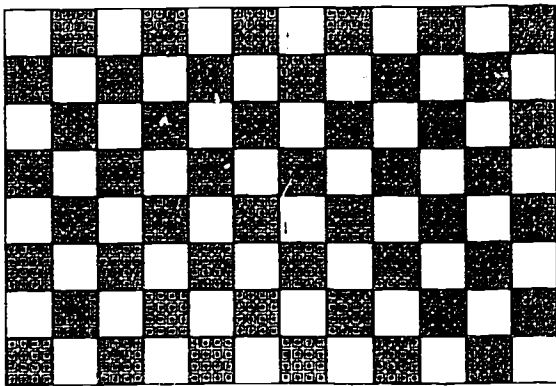
This is how garl k painted $1/2$ of the box.



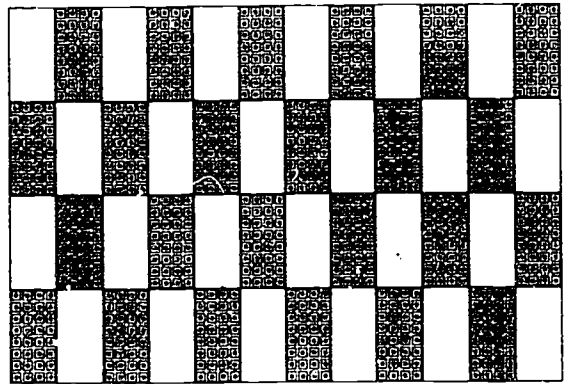
This is how michael k painted $1/2$ of the box.



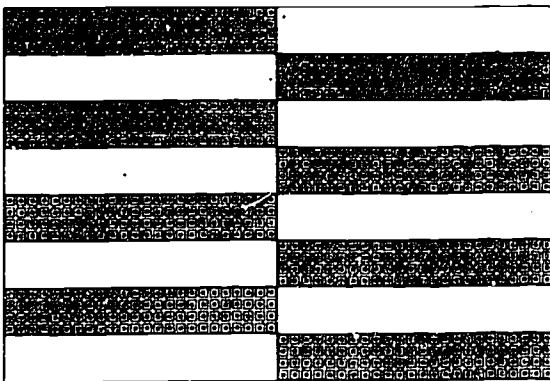
This is how raymond h painted 1/2 of the box.



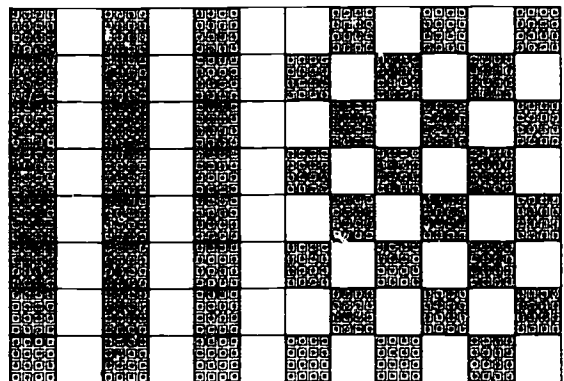
This is how lynn e painted 1/2 of the box.



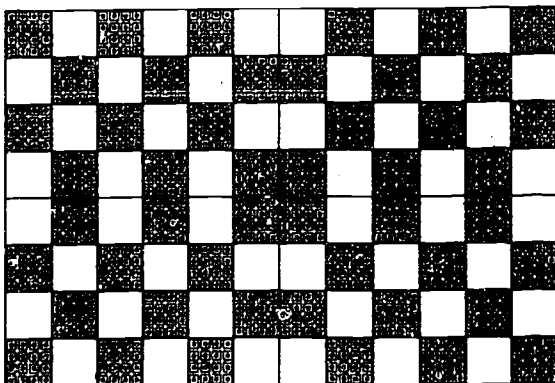
This is how david c painted 1/2 of the box.



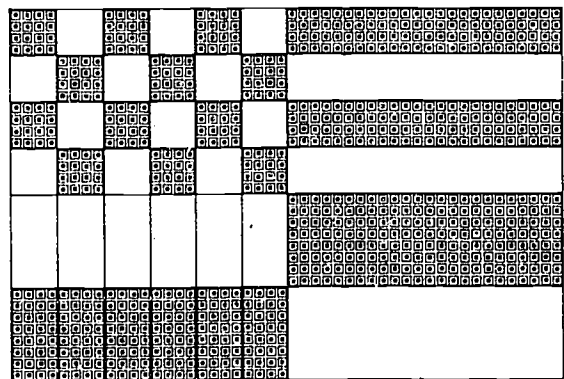
This is how denise h painted 1/2 of the box.



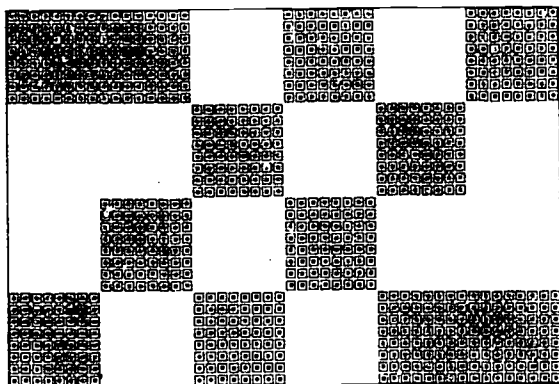
This is how mary e painted 1/2 of the box.



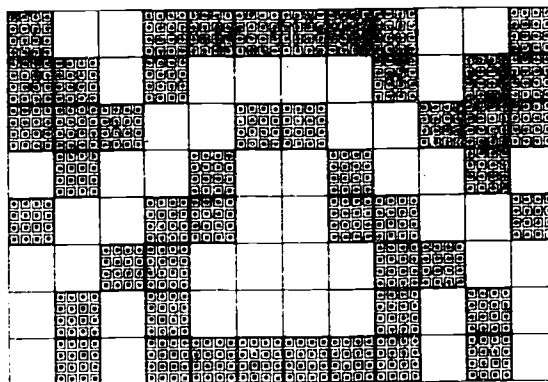
This is how fred d painted 1/2 of the box.



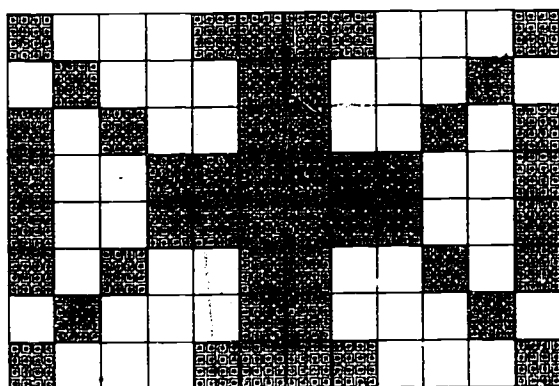
This is how rebecca m painted 1/2 of the box.



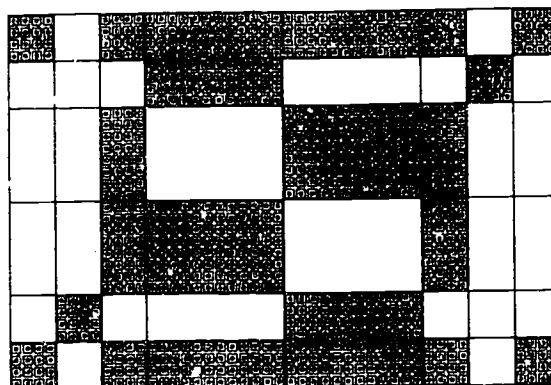
This is how frederic m painted 1/2 of the box.



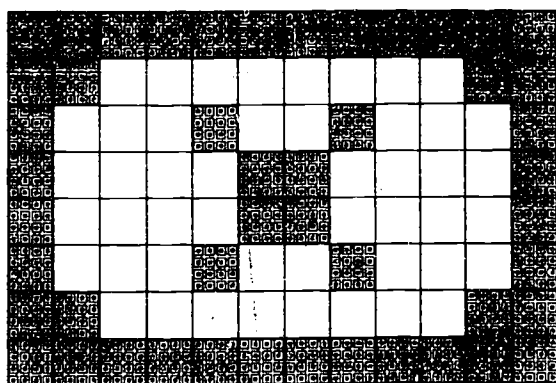
This is how teresa m painted 1/2 of the box.



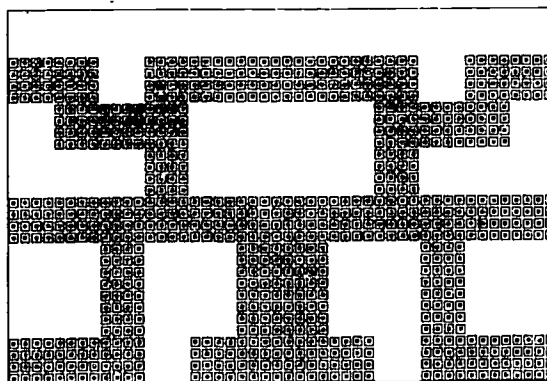
This is how joseph h painted 1/2 of the box.



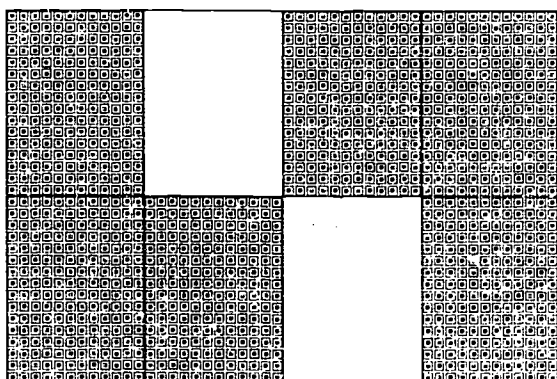
This is how fred s painted 1/2 of the box.



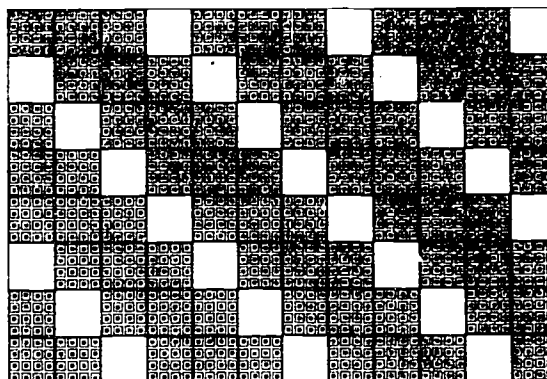
This is how jeffrey c painted 1/2 of the box.



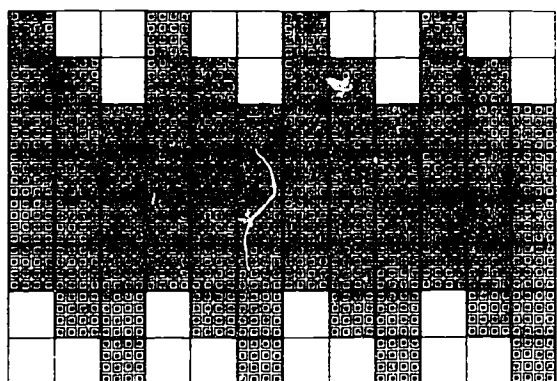
This is how rogelio painted $\frac{6}{8}$ of the box.



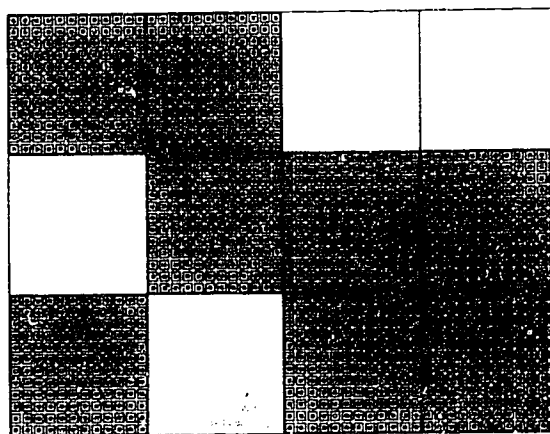
This is how tashua p painted $\frac{6}{8}$ of the box.



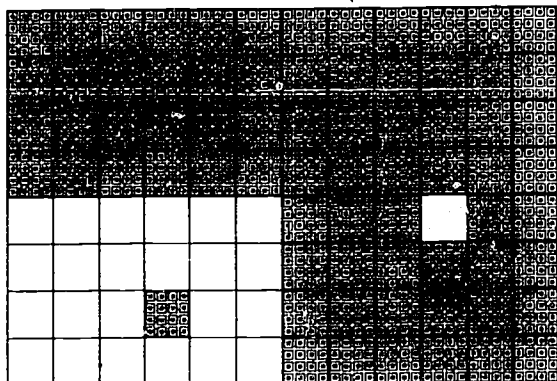
This is how mana t painted $\frac{6}{8}$ of the box.



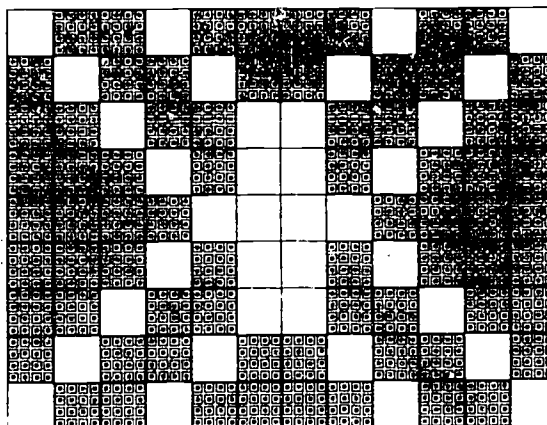
This is how ellen b painted $\frac{2}{3}$ of the box.



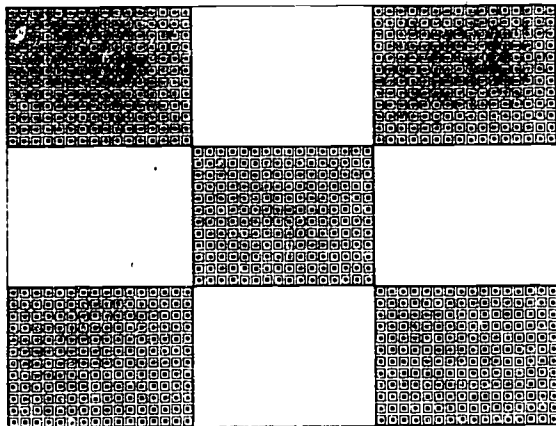
This is how john t painted $\frac{6}{8}$ of the box.



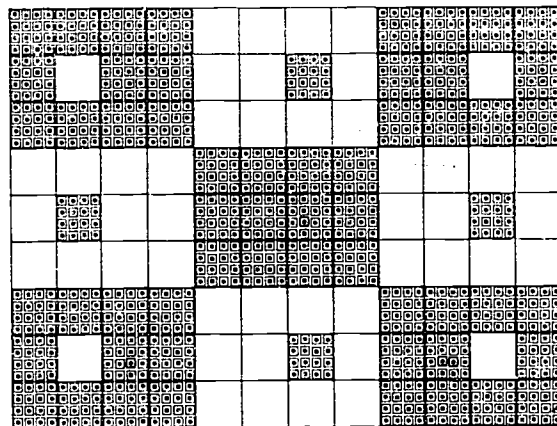
This is how edward m painted $\frac{2}{3}$ of the box.



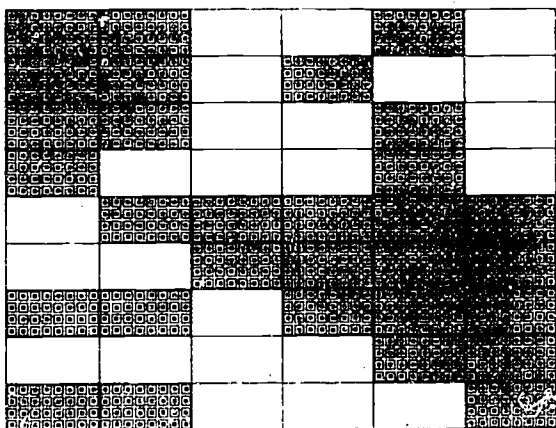
This is how kevenl i painted $5/9$ of the box.



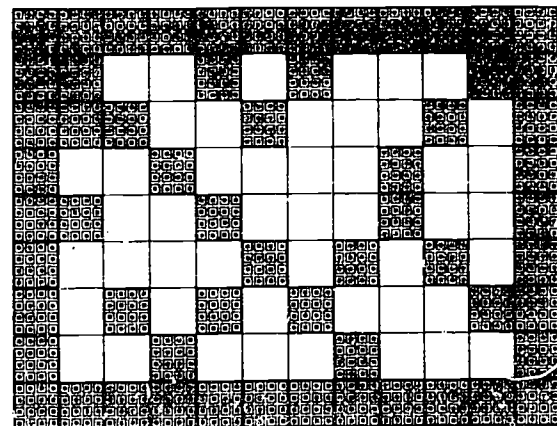
This is how lawston painted $5/9$ of the box.



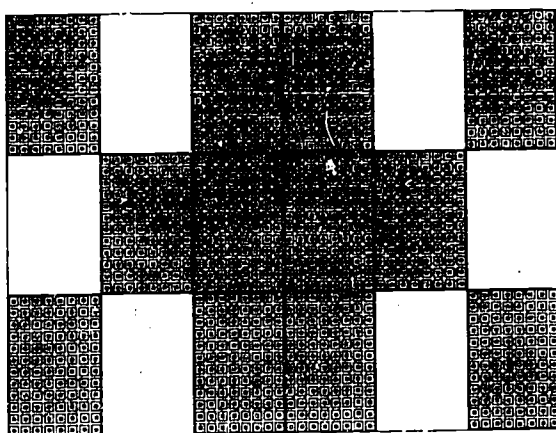
This is how russell g painted $5/9$ of the box.



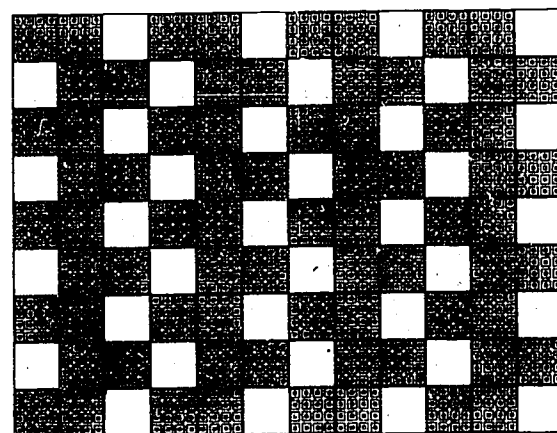
This is how leslie w painted $5/9$ of the box.



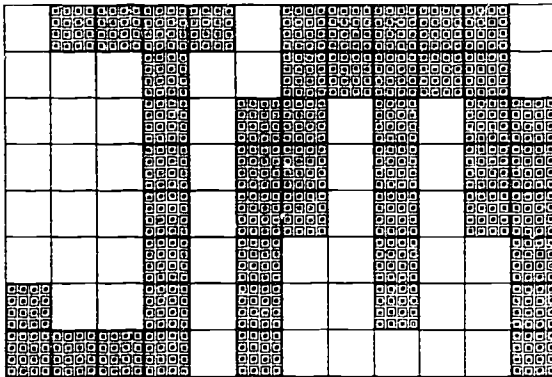
This is how mara t painted $2/3$ of the box.



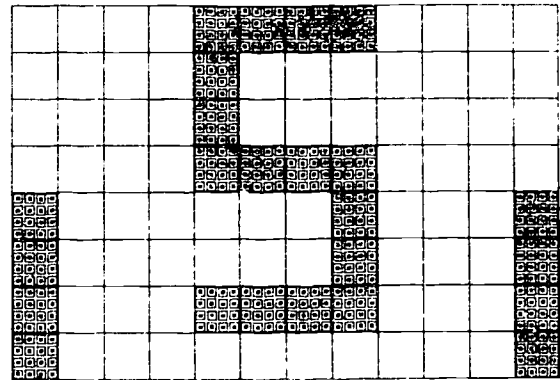
This is how robert s painted $2/3$ of the box.



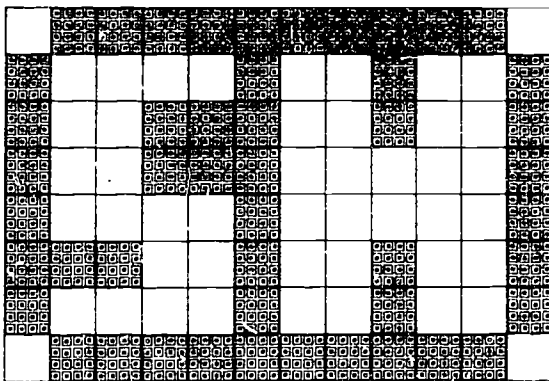
This is how john m painted 1/2 of the box.



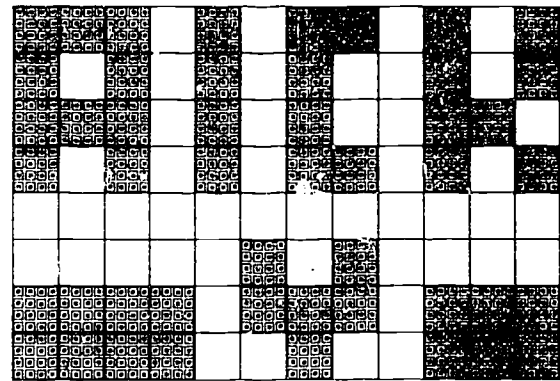
This is how stephen r painted 1/4 of the box.



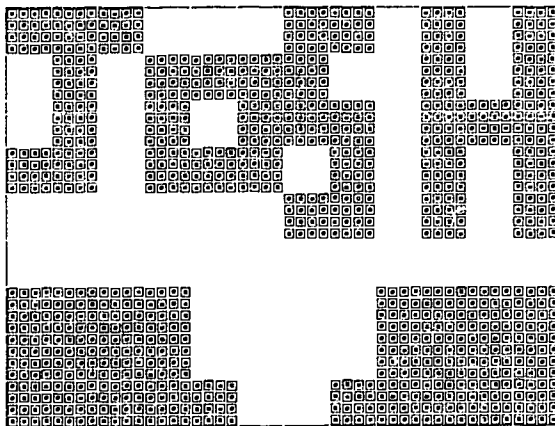
This is how susan h painted 1/2 of the box.



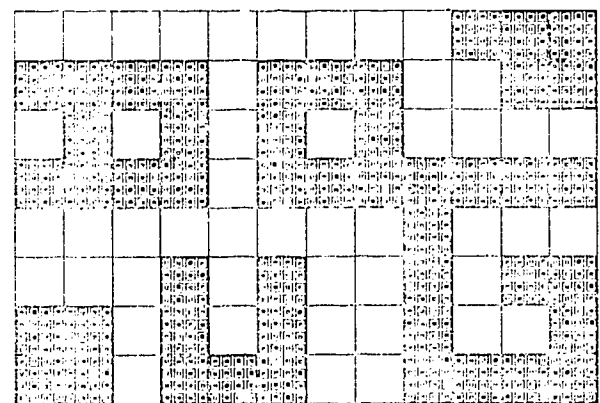
This is how richard painted 1/2 of the box.



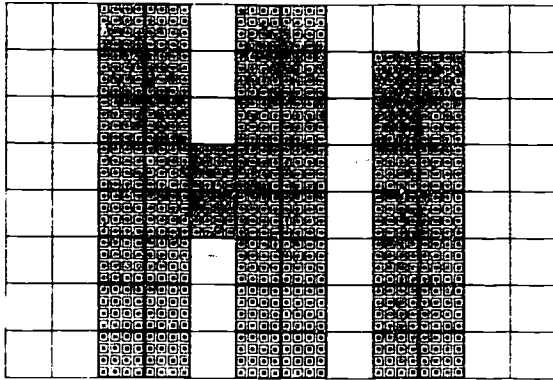
This is how joshua painted 5/9 of the box.



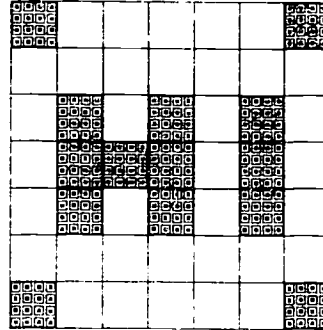
This is how douglas painted 1/2 of the box.



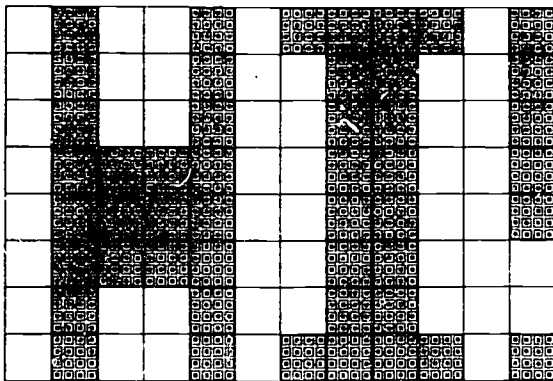
This is how hermond h painted 1/2 of the box.



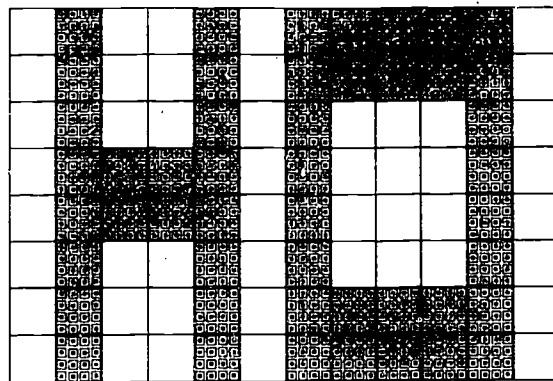
This is how margaret s painted 1/2 of the box.



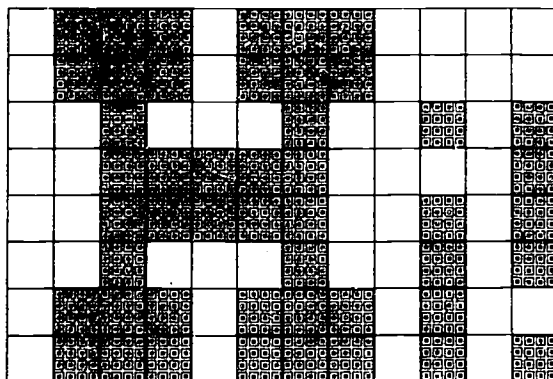
This is how eric m painted 1/2 of the box.



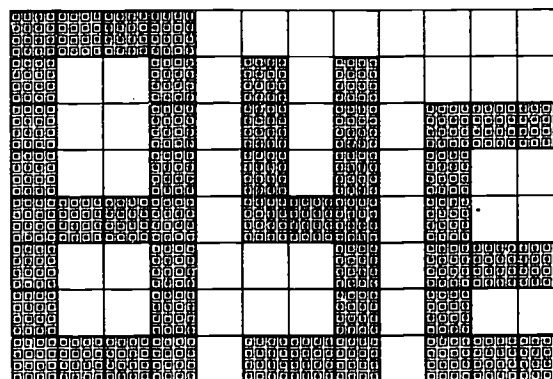
This is how theodore p painted 1/2 of the box.



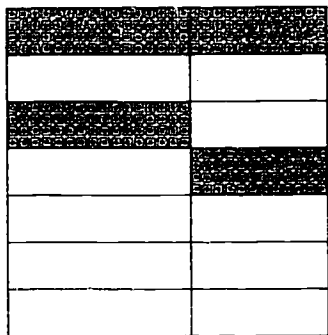
This is how derek k painted 1/2 of the box.



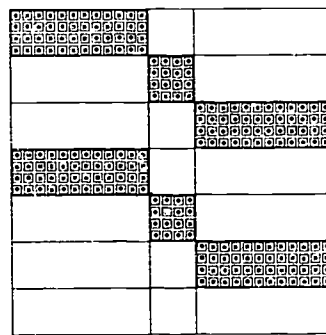
This is how christine painted 1/2 of the box.



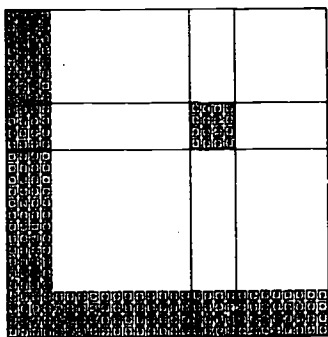
This is how lesli g painted 2/7 of the box.



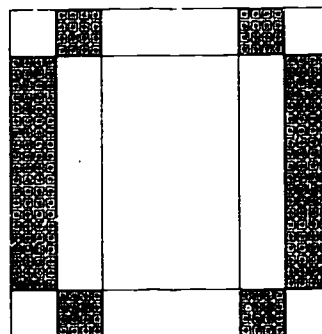
This is how lisa painted 2/7 of the box.



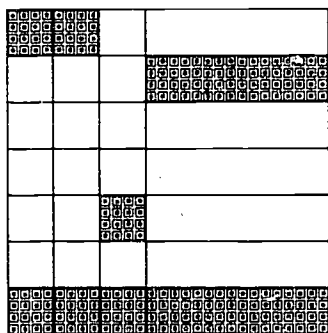
This is how laura m painted 2/7 of the box.



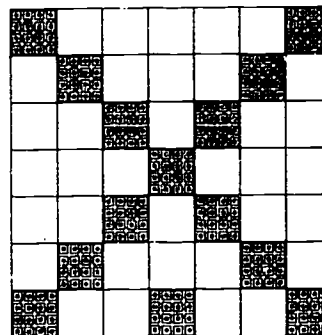
This is how darrell d painted 2/7 of the box.



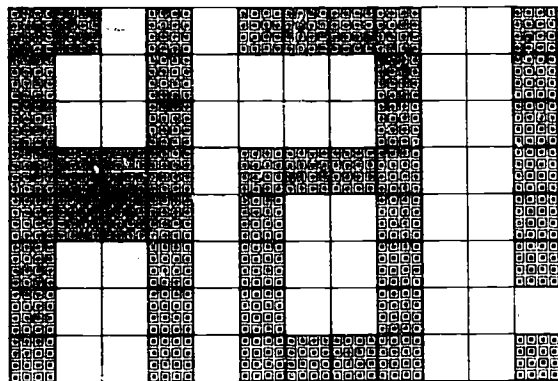
This is how marshall painted 2/7 of the box.



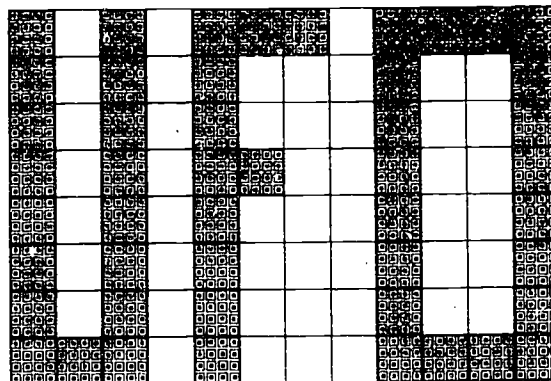
This is how william b painted 2/7 of the box.



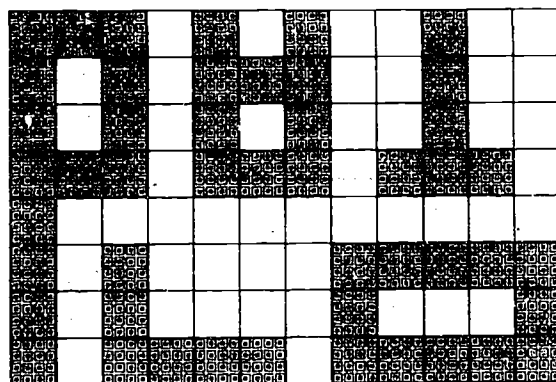
This is how david k painted 1/2 of the box.



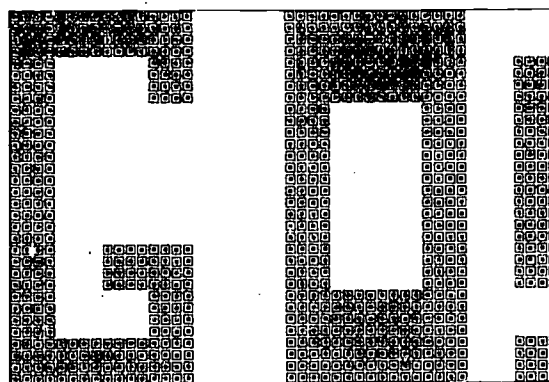
This is how paul p painted 1/2 of the box.



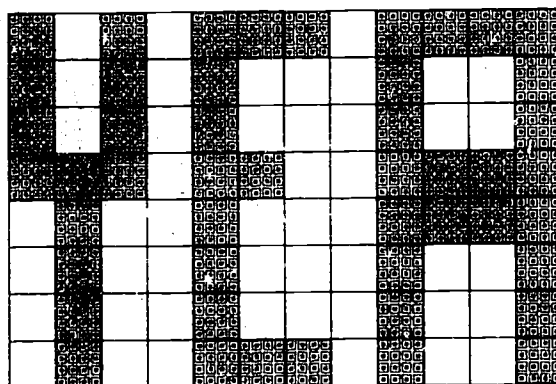
This is how derek k painted 1/2 of the box.



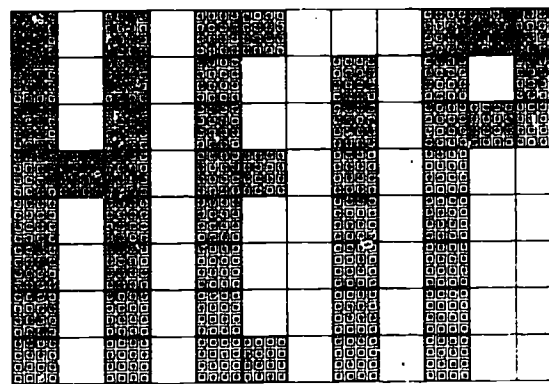
This is how jeffrey s painted 1/2 of the box.



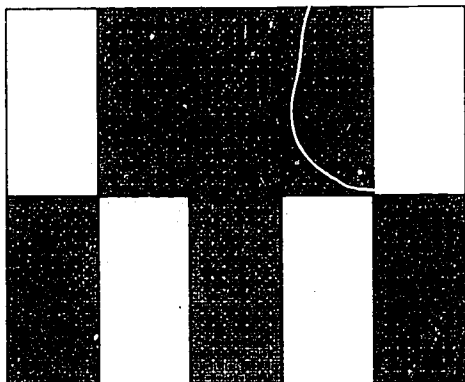
This is how peter j painted 1/2 of the box.



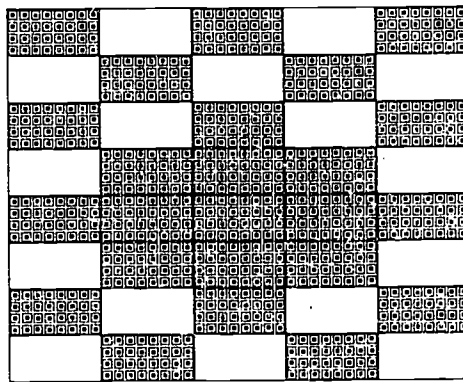
This is how david b painted 1/2 of the box.



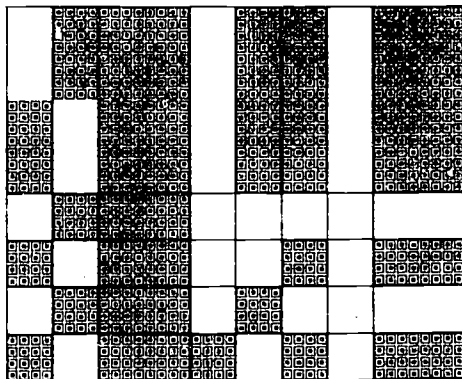
This is how amy h painted 3/5 of the box.



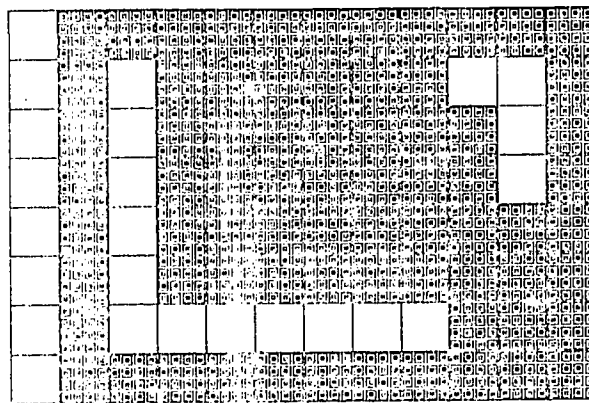
This is how Tracy painted 3/5 of the box.



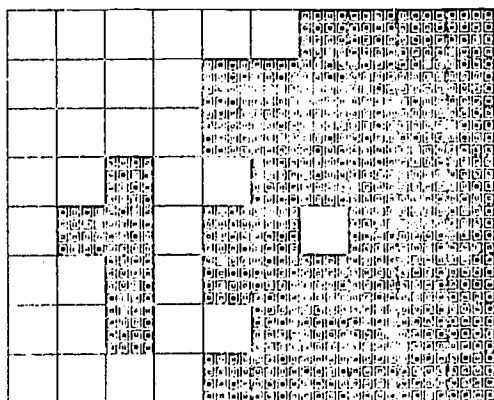
This is how fred d painted 3/5 of the box.



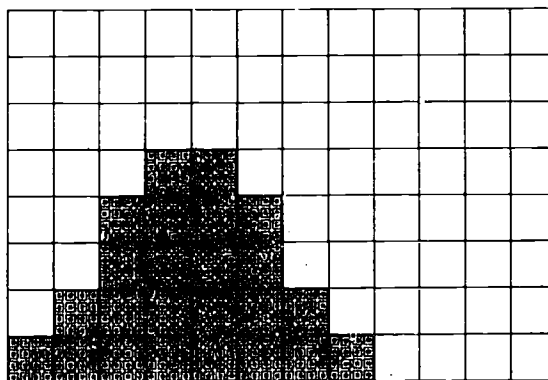
This is how langford painted 6/8 of the box.



This is how kevin painted 3/5 of the box.



This is how daniel s painted 1/4 of the box.

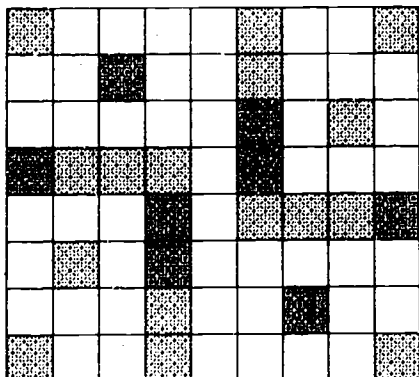


Our experience has been that a large number of students participating in a library greatly increases the richness of students' work. This lesson was first tried with only four students, and although they were described as bright, creative students from an open classroom environment, their paintings were quite unimaginative. Each student did some paintings and looked at the paintings done by the other three, but new ideas were few and enthusiasm was only mild. However, during a three-week period when 300 students used the lesson, it exploded with new ideas as students looked at each other's work and responded with their own.

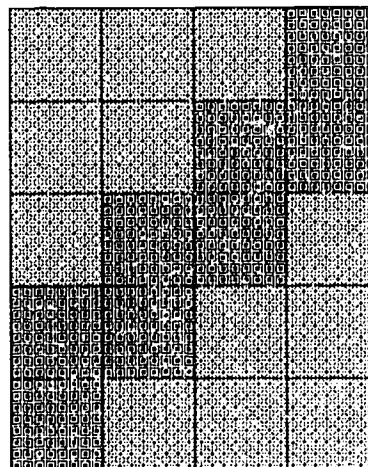
This lesson and others like it provide strong motivation for students to engage in tasks which require them to relate mathematical concepts to concrete models. Students are observed to engage in many lines of mathematical thought when creating a painting. For example, if the student has chosen $\frac{3}{5}$ as the fraction to represent, and in doing a creative pattern has mistakenly painted $\frac{13}{20}$ too much, PLATO will respond, "You have painted $\frac{13}{20}$ of the box". The student must then decide whether that is too much or too little and how much paint to add or remove. Some students will count same-size pieces and possibly use equivalent fractions. Others will visualize moving painted parts around to make an equivalent simpler painting. Still others may partition the box and deal with the parts separately, deciding whether $\frac{3}{5}$ of each part is painted.

Later in the curriculum is the "Paint Addition Library". Here the student can use two kinds of paint to illustrate an addition problem. Students can ask PLATO for a problem to illustrate or make one of their own. In either case, when the student has correctly painted both of the addends, he must tell how much of the box is painted altogether. This may be done by several methods, including drawing lines on the box until it is divided into some number of equal pieces, each of which is either all painted or all unpainted. (This is a visual method of finding a common denominator for the two fractions.) Following are some students' representations of various additions.

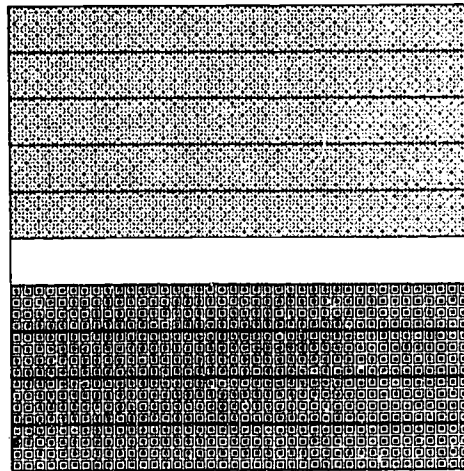
This is derek k's painting: $\frac{8}{72} + \frac{16}{72} = \frac{24}{72}$



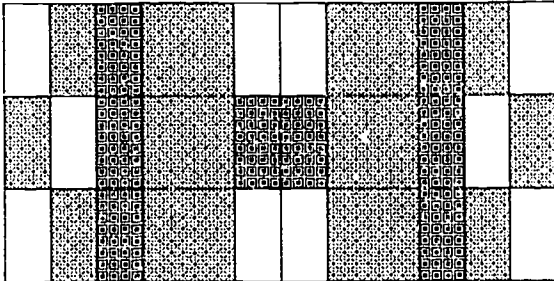
This is judith's painting: $\frac{8}{28} + \frac{12}{28} = 1$



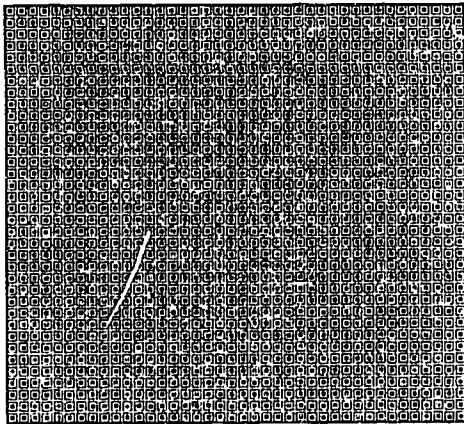
This is ruth's painting. $2/5 + 1/2 = 9/10$



This is maggie's painting: $2/9 + 1/2 = 26/36$

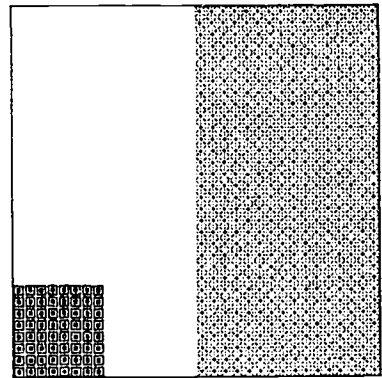


This is ed's painting: $1/1 + 8/1 = 1/1$

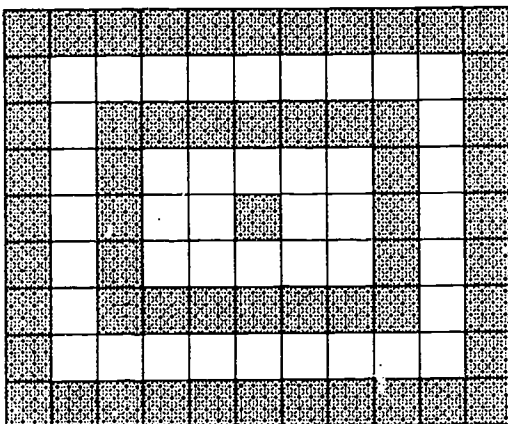


This is ethan's painting:

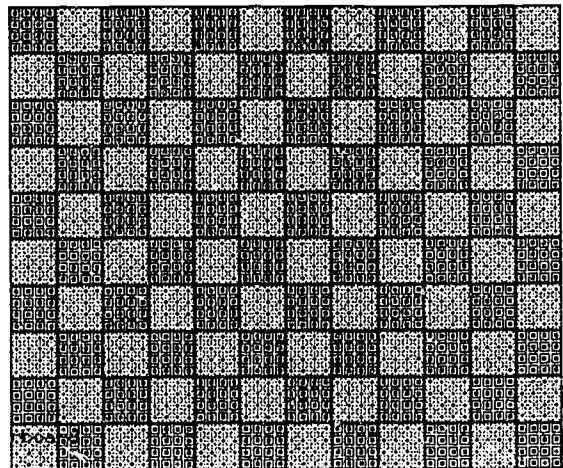
$$1/16 + 1/2 = 9/16$$



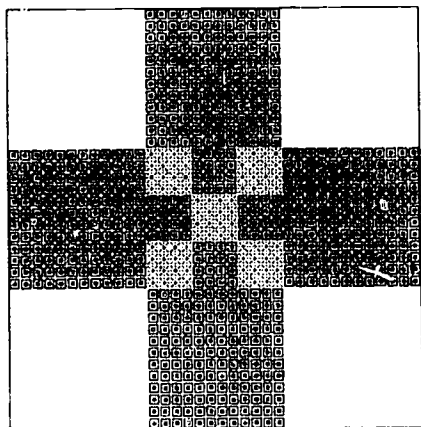
This is christi's painting. $8/99 + 57/99 = 57/99$



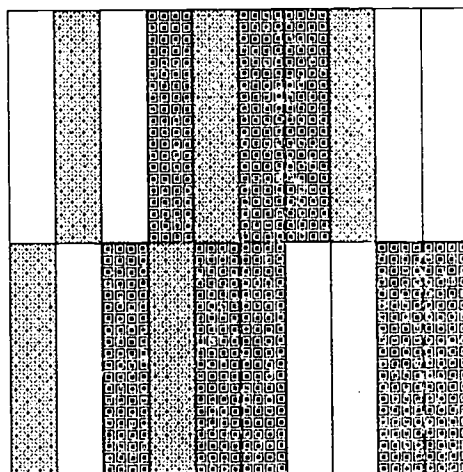
This is faline's painting. $1/2 + 1/2 = 1$



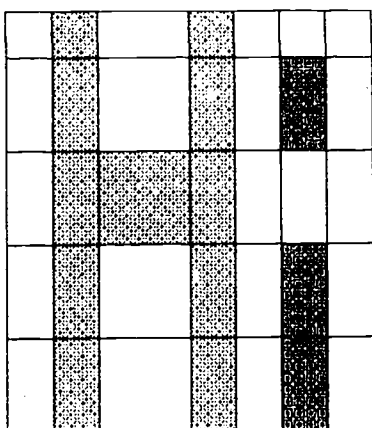
This is ducky's painting: $40/61 + 5/61 = 5/9$



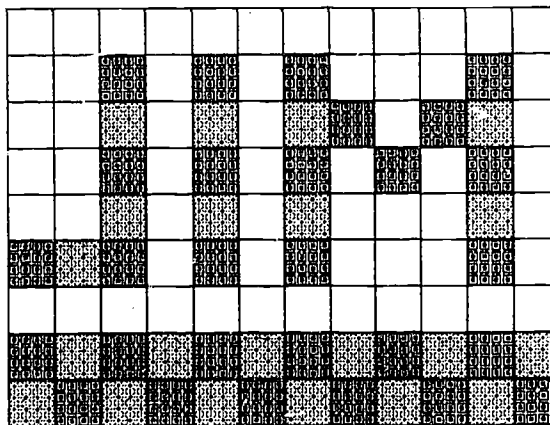
This is cathy's painting: $2/5 + 1/4 = 13/20$



This is douglas's painting: $6/72 + 22/72 = 28/72$



This is jim's painting: $28/188 + 21/188 = 49/188$



"Skywriting and Spider Web" is a library lesson that lets the student write a "program" to move an airplane or a spider around the screen. (See Figure I-1.) The airplane leaves a trail of smoke, and the spider leaves a web, so that the student's program makes a pattern on the screen. The simple programming language has three commands:

"f" to go forward (specify how many units)

"t" to turn (specify what fraction of a turn)

"r" to repeat the program

There is also an option to turn the smoke or web off and on.

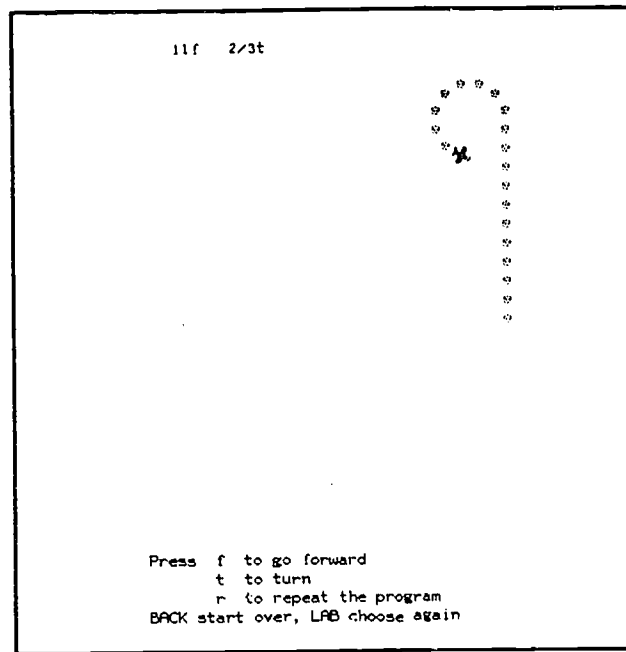
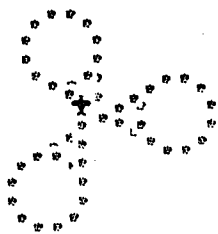


Fig. I-1. Writing a program in the "Skywriting and Spider Web" library. The airplane has been instructed to go 11 units forward, then turn $2/3$ of a full circle. The program is recorded at the top of the screen as it is written. The key to the simple programming language is shown at the bottom of the screen.

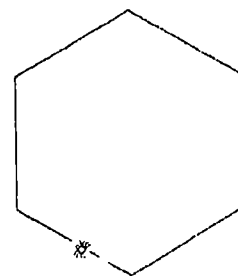
When the student has finished writing a program, he has the option of giving the resulting picture a title and saving it in the library. The following pages show some programs from the library. In each frame the student's program appears at the top of the screen. At the bottom of the screen is the student's name and the title he has chosen for the picture. Each student has his own special space in the library to save a program. He may replace his program with a new one whenever he chooses.

This lesson gives the student experience with a simple programming language, with iteration, and with the idea of a correspondence between symbols and pictures. In addition, it applies fractions to the "turn", a fraction model very different from the common ones such as regions and discrete sets. There is also a great wealth of geometric content that can be pursued. For example, the spider's turns are spot rotations, while the airplane's turns are arcs of circles. One interesting exercise is to explore the difference this makes by running the same program with the spider, then with the airplane. Another possibility is to explore various "classes" of programs; e.g., programs of the form " n forward, $1/m$ turn, repeat" always make a regular polygon with m sides of length n .

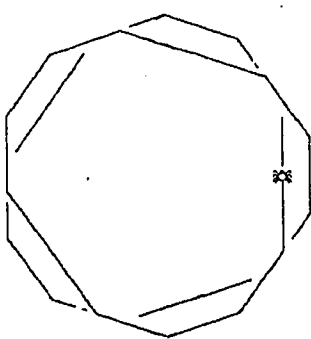
3f 0/3t 7f 0/3t 8f 0/3t 4f



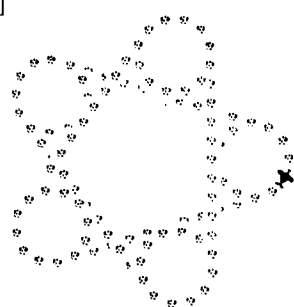
cheryl -- "3 leaf clover"

2f 1/2t 1/3t 5f r
↑

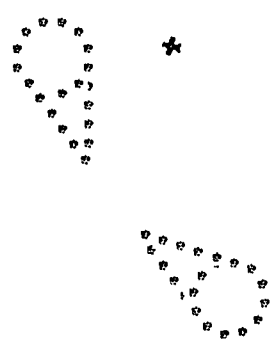
laura -- "Hexagon"

4f 1.1t 4f 1.1t 4f 1.1t 4f 1.1t 4f r
↑

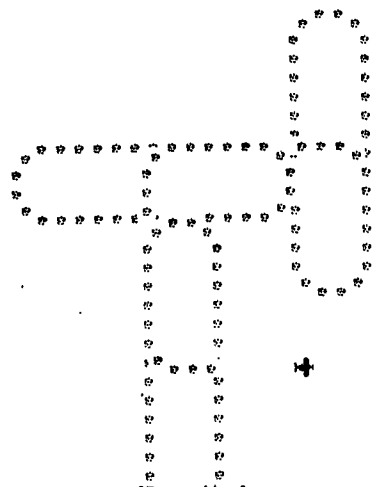
kevin -- "nimnul"

11f 12/5t r
↑

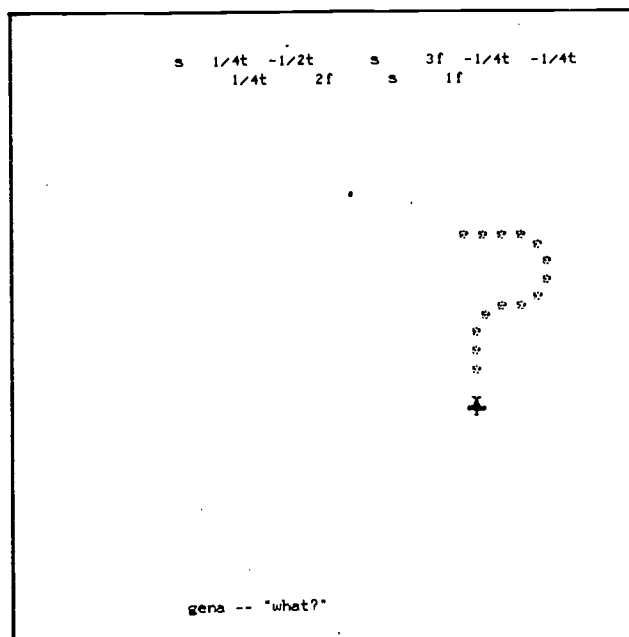
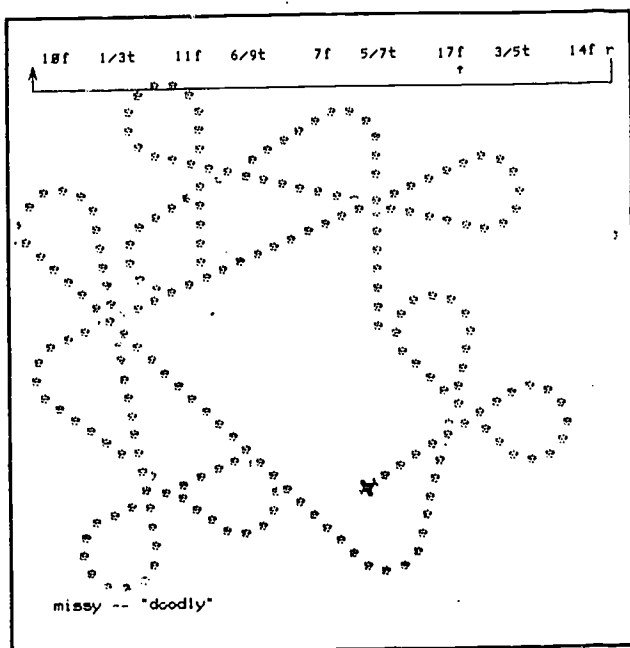
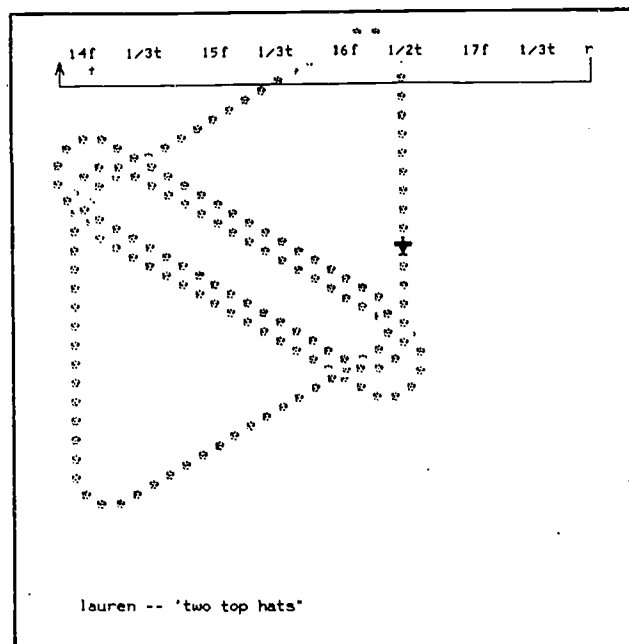
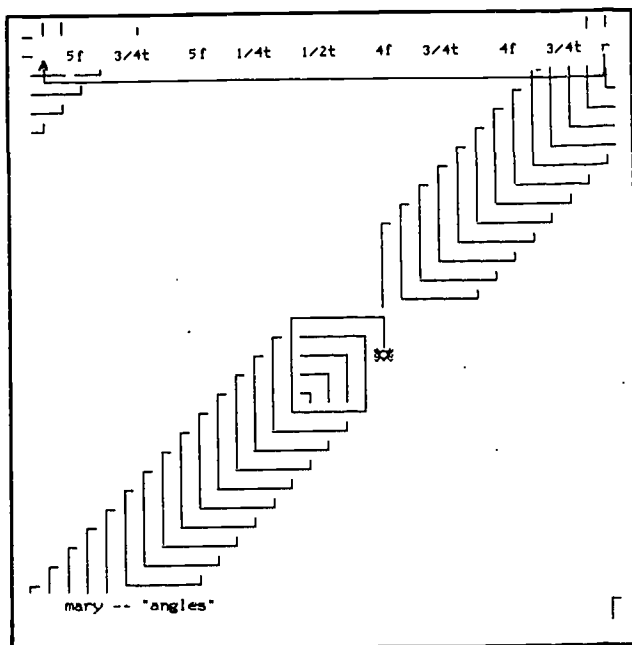
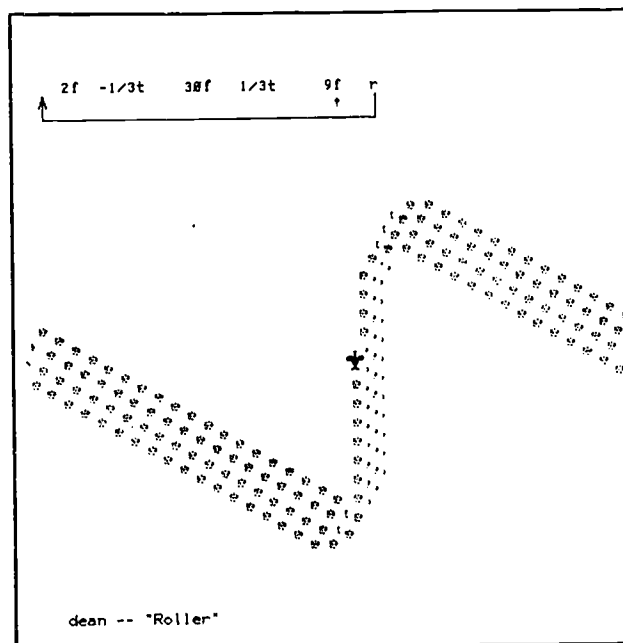
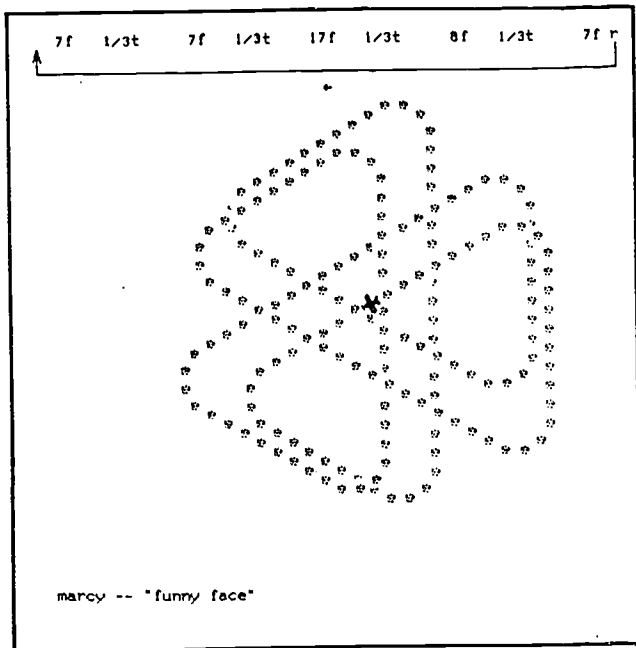
lori -- "circle star"

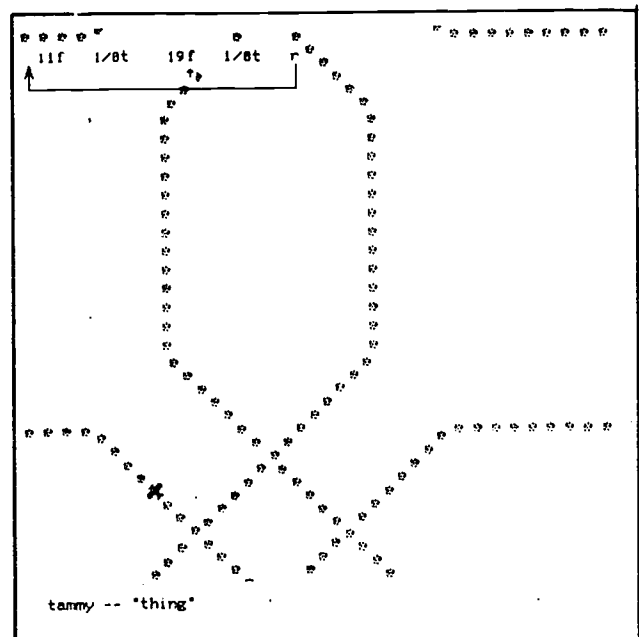
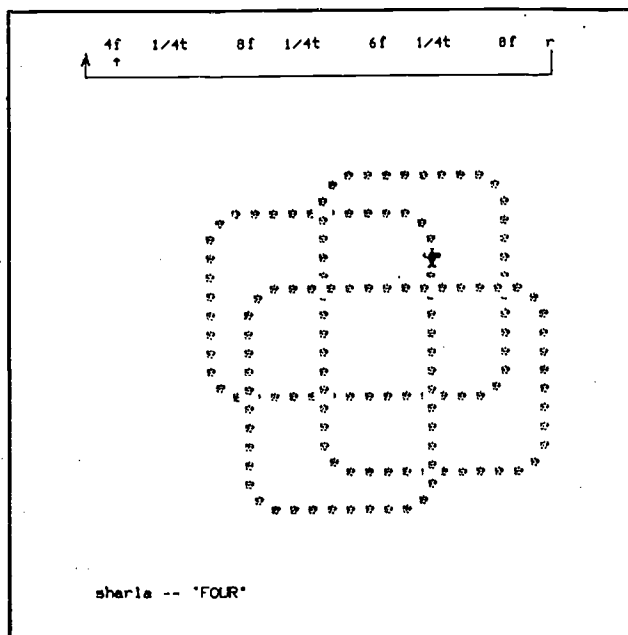
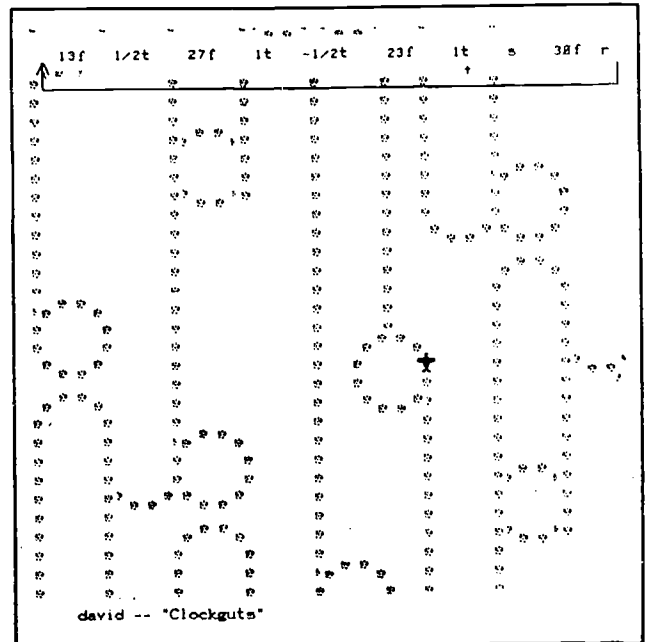
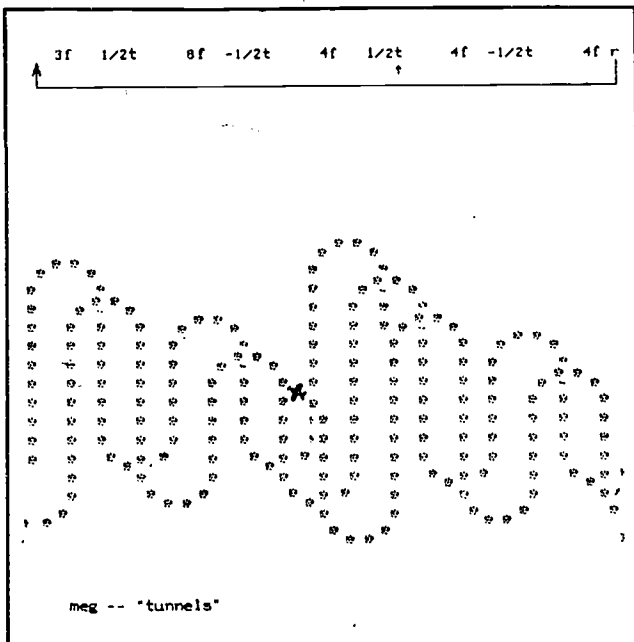
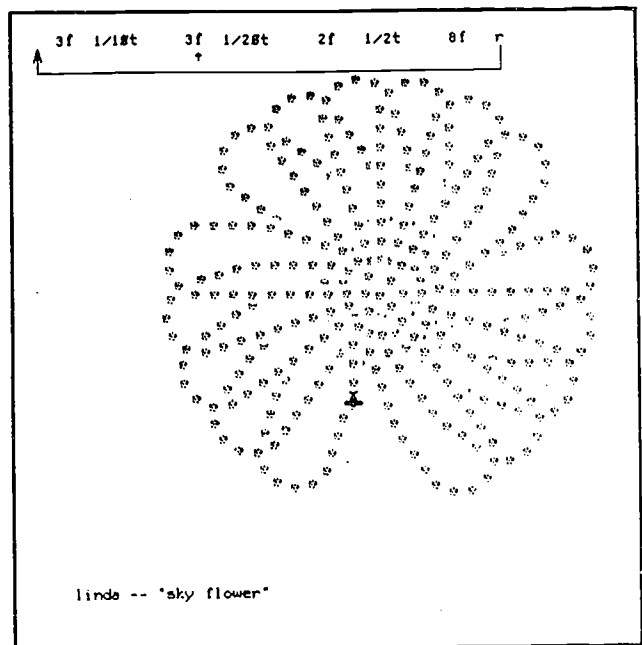
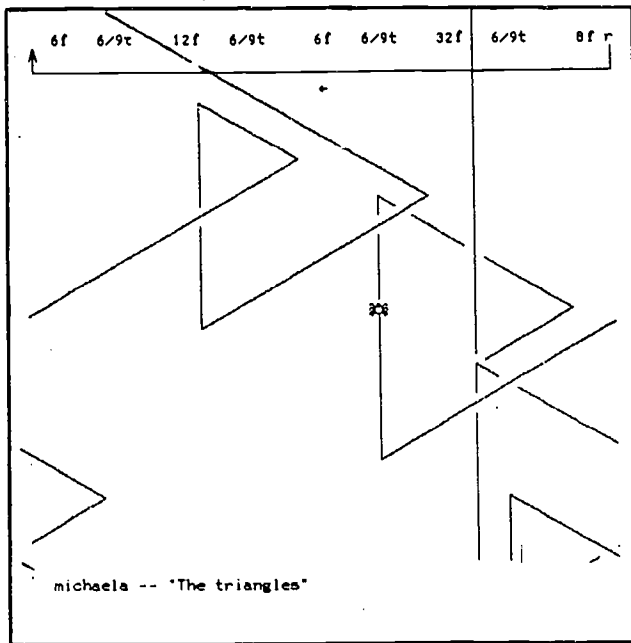
6f 1.6t 6f s 4f s 6f
1.6t 6f s 31f

marivel -- "maarriivveell"

9f 1/2t 11f 1/2t 4f 2/8t 2f s 4f r
↑

elaine -- "Paperclips"





The network characteristics of the PLATO computer system allow students to communicate not only with the computer, but with each other. This can have large educational payoffs. Students generate and share a wide variety of approaches to what is traditionally very stereotyped subject matter. The richness of the exchange is enhanced by the fact that the sharing takes place not merely across students, but across classes, schools, and even communities. Far from being a mere drill-and-practice robot, the computer can be a medium for social interaction, mathematical discovery, and aesthetic expression.

Appendix II

Other Uses of the PLATO Fractions Materials

It is recommended that the PLATO fractions curriculum be used on a daily basis as an integral part of the classroom mathematics program. However, it is the case that the lessons have also been successfully used separately as supplementary activities for students who do not have access to PLATO regularly enough for it to be a major part of their mathematics program. The game-type lessons are often used this way to provide motivating practice with skills or exploratory experience with new concepts. For example, in a local school that did not have PLATO terminals, several classes arranged parent carpools to take the class to a PLATO classroom at the University of Illinois for an hour's PLATO work twice each month. Such infrequent use of the PLATO curriculum would have made it difficult to deliver an organized PLATO sequence that paralleled classroom mathematics instruction. Instead, these students were assigned games and other activities that related to present class activities, but that were not necessarily related to previous PLATO work. Teachers, students, and parents all felt that the time spent was rewarding and educationally valuable.

In addition, these lessons have been included in a curriculum package for adult students who need instruction in basic mathematics.² Along with a wide selection of other materials relevant to the needs of adult basic

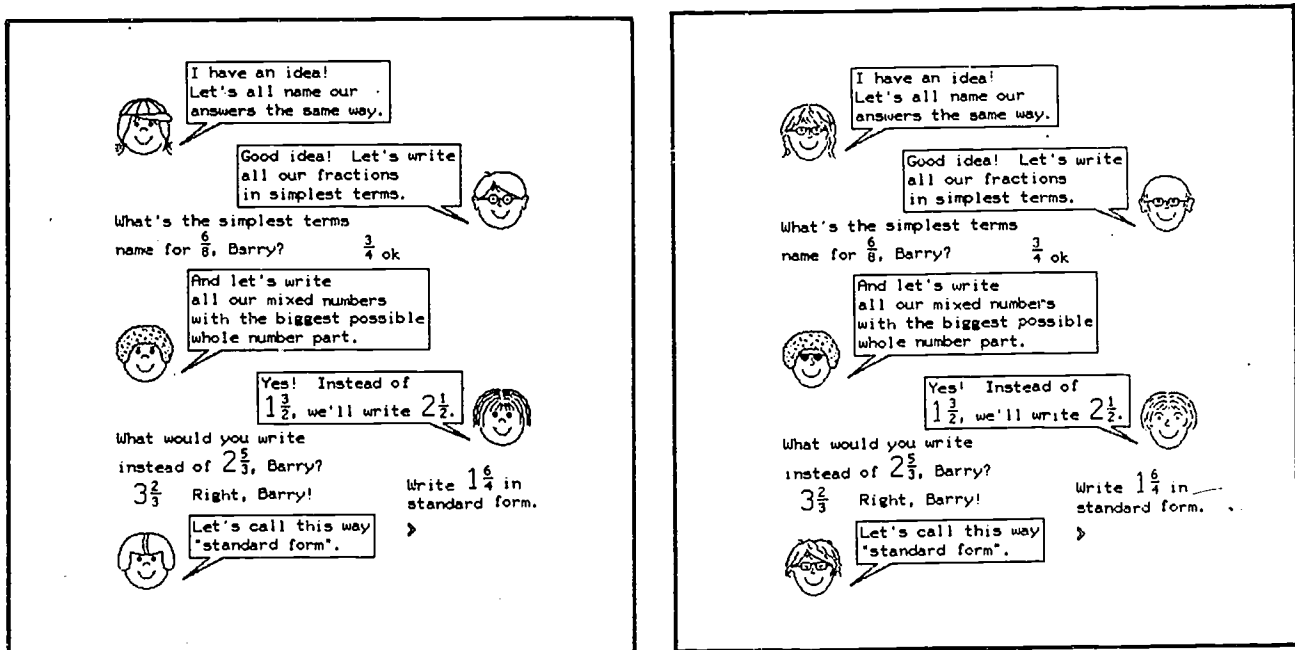


Fig. II-1. Two screen displays from lesson "Simplifying Answers to Addition Problems". The first display shows the lesson as it appears for elementary school students. The second is the same display, but with the more mature faces as used with older students.

²Siegel, M. A., et al., "PLATO Computer-based Education in Prisons", *American Journal of Correction*, Vol 40, No 1, January-February 1978.

education students, these lessons are organized into topics that can be assigned by the instructor. For these older students an alternate set of characters was added to many of the lessons to make them more appropriate for adult learners. (See fig. II-1.) This instructional package has found enthusiastic acceptance among students in correctional institutions and other facilities serving educationally disadvantaged adults.

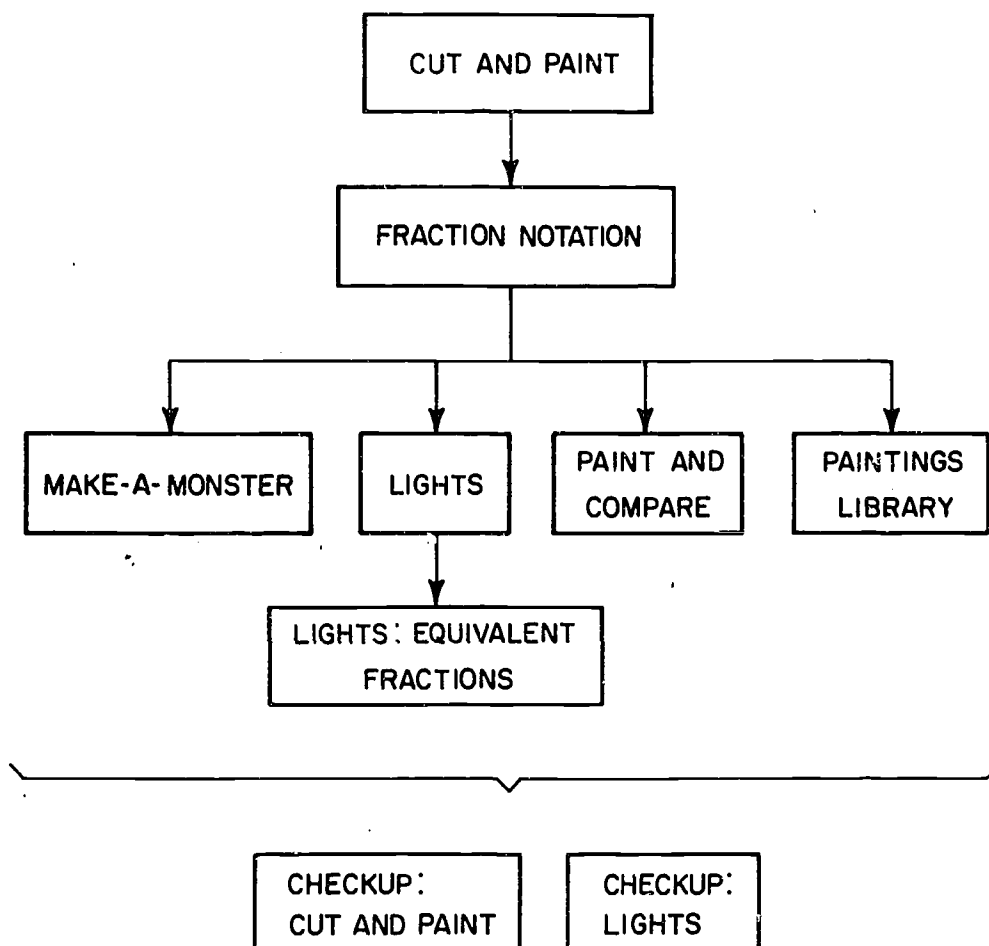
Appendix III Instructional Modules

The lessons within the fractions curriculum are organized into modules for use with the management system in a classroom mathematics program. The modules listed here are those that have been most often used over the past years. Many other modules have been created to meet specific needs of individual students or classes. The ones described here have been found appropriate for most students.

Each module has a topic of major emphasis, but also includes lessons which provide review of previous material, readiness for later material, and general experience and enrichment. In addition to the lessons that are shown on these module charts, the student receives review and general experience material chosen by PLATO based on information the teacher has supplied about the student's mathematical background. In each PLATO session, the student may see several lessons -- some new and some continued from previous sessions.

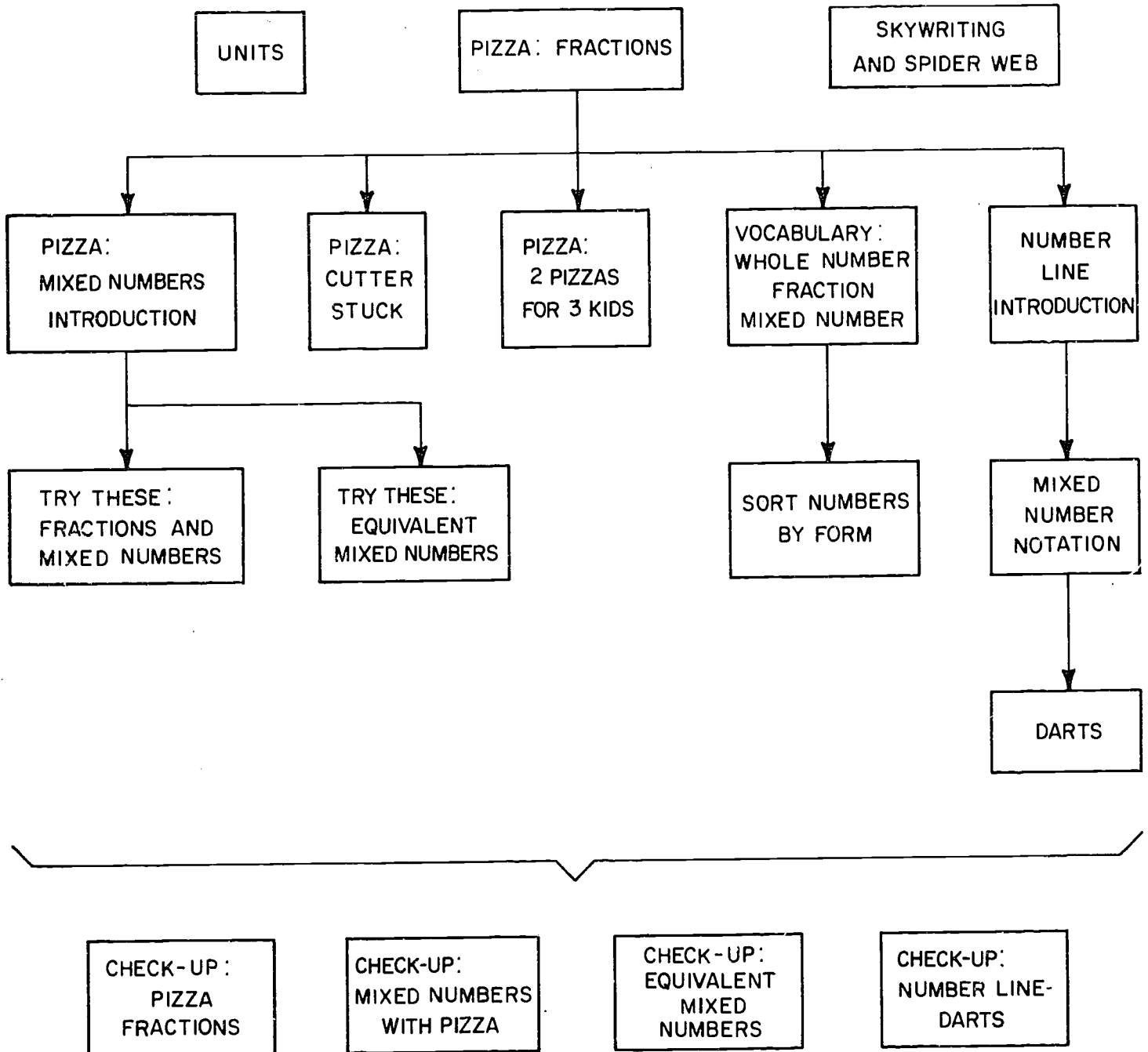
MEANING OF FRACTIONS

a simplified flow chart



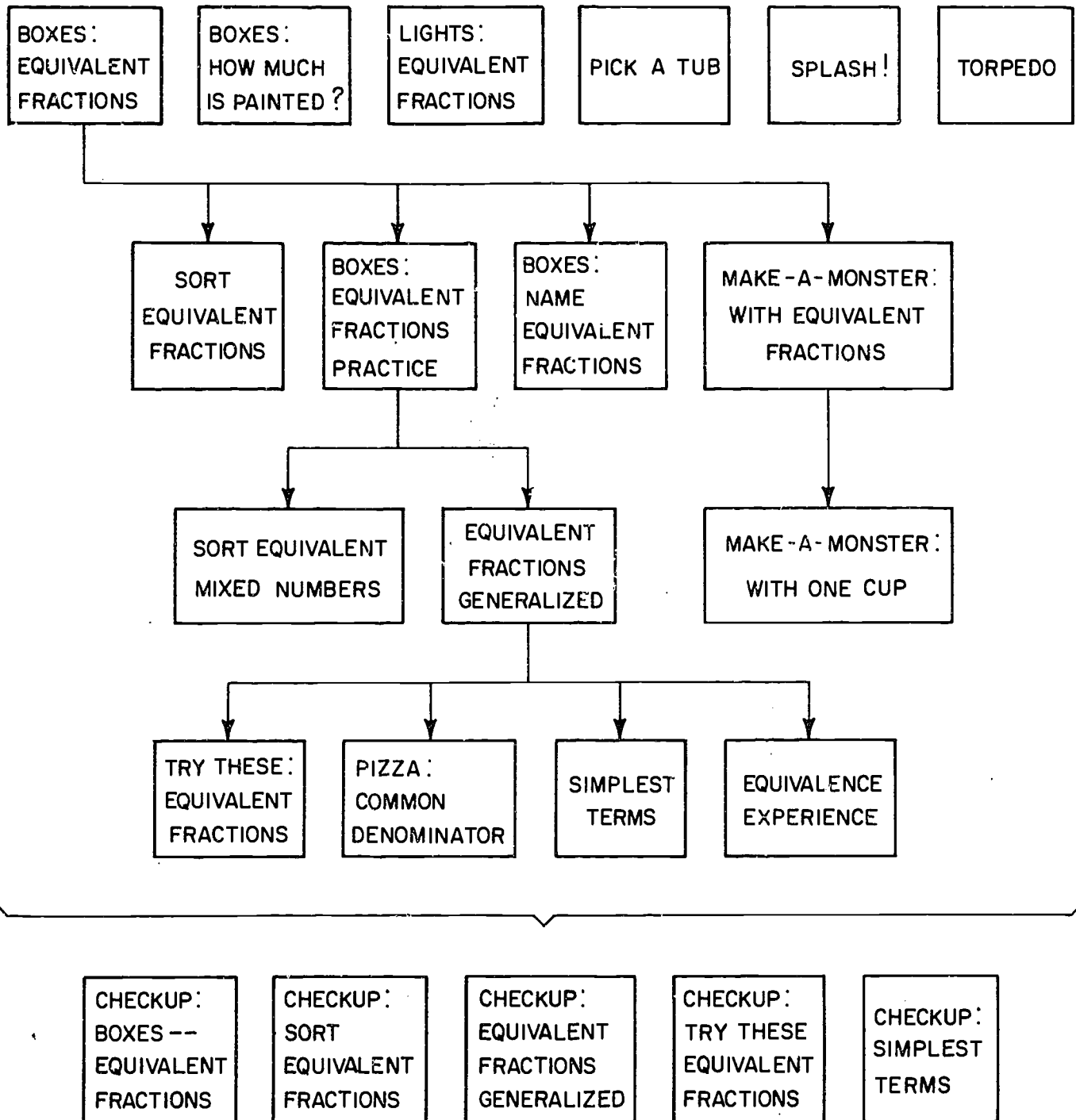
MIXED NUMBERS and FRACTIONS ≥ 1

a simplified flow chart



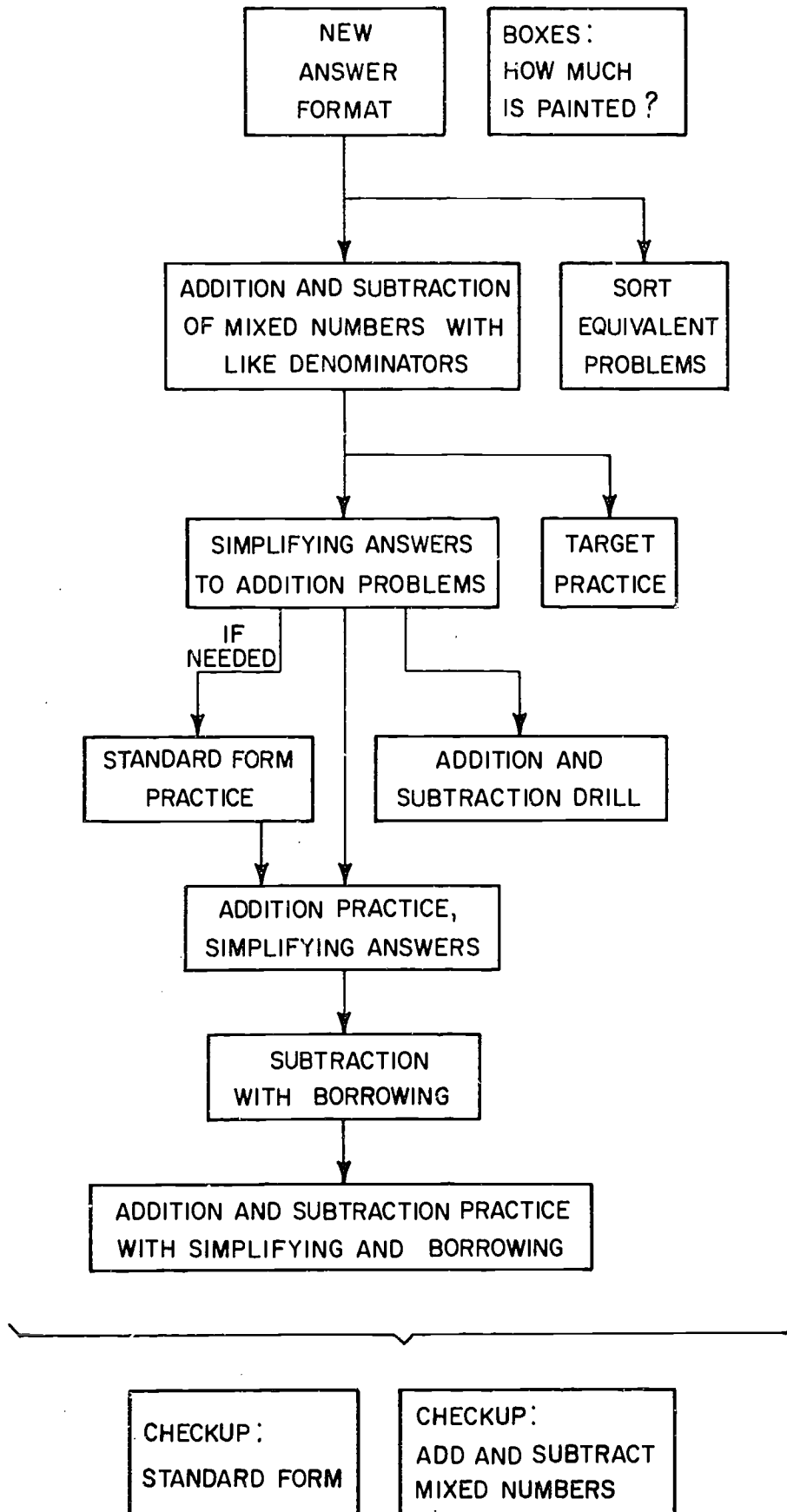
EQUIVALENT FRACTIONS

a simplified flow chart

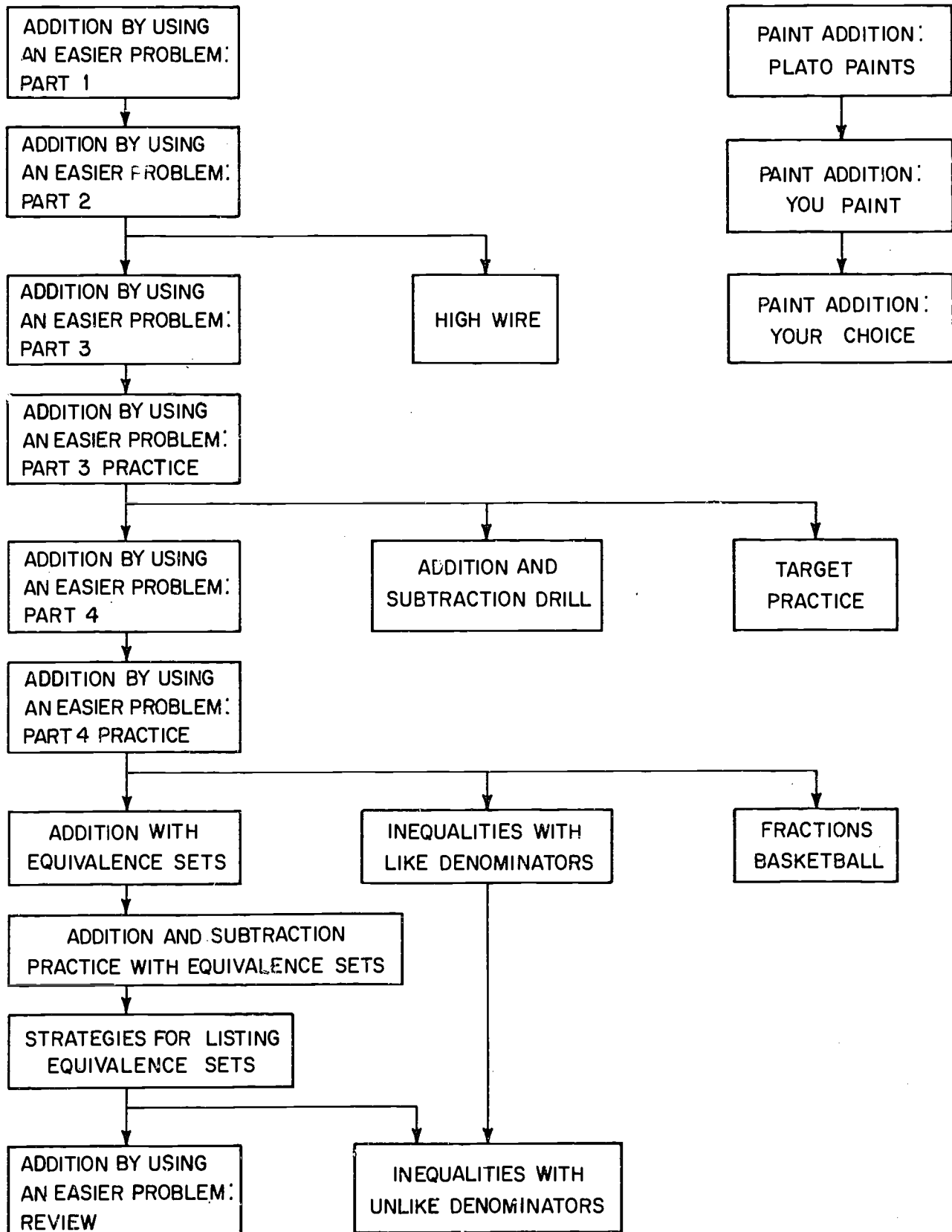


ADD AND SUBTRACT MIXED NUMBERS WITH LIKE DENOMINATORS

a simplified flow chart

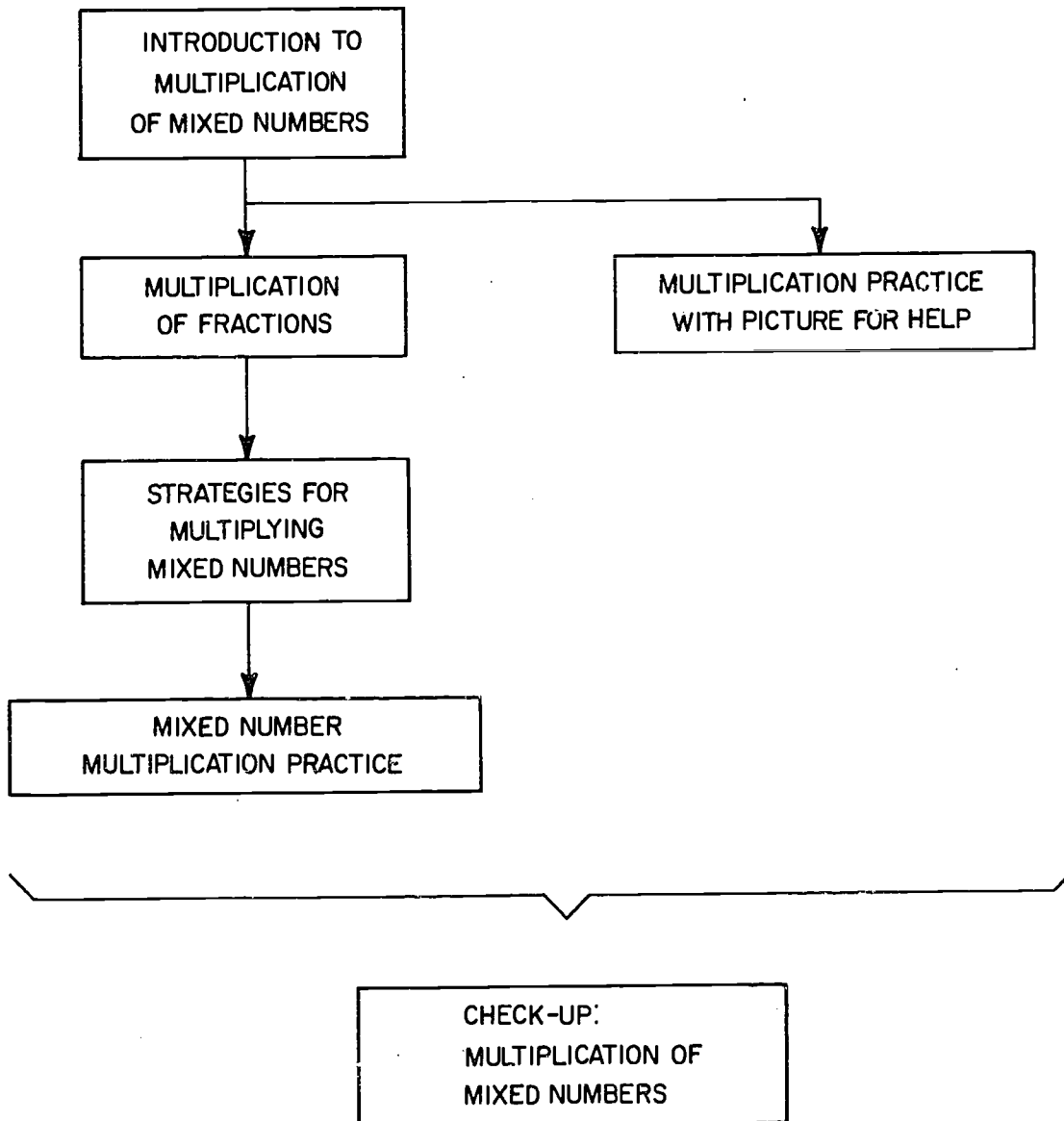


ADD AND SUBTRACT MIXED NUMBERS WITH UNLIKE DENOMINATORS a simplified flow chart

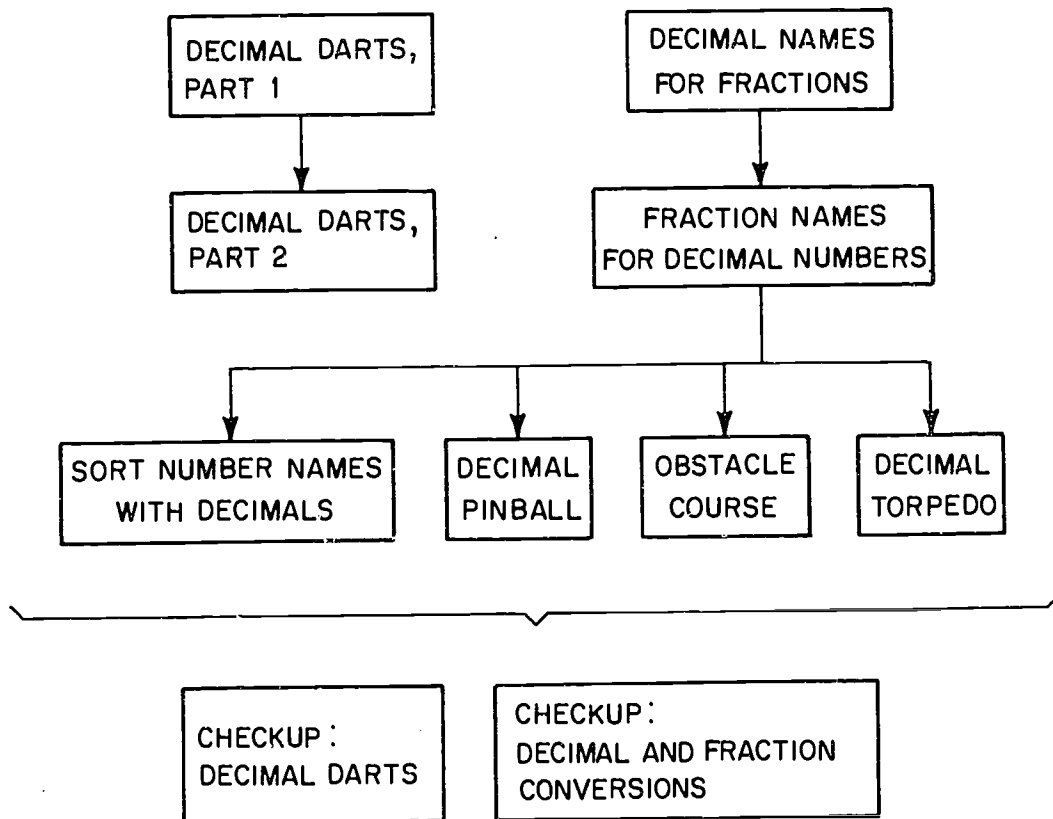


MULTIPLICATION OF MIXED NUMBERS

a simplified flow chart



MEANING OF DECIMAL FRACTIONS
a simplified flow chart



Appendix IV
Some Related Publications

- Dugdale, S., and Kibbey, D., Supplementary Materials for the Fractions Curriculum, CERL, September 1975.
- Dugdale, S., and Kibbey, D., Programs from the Skywriting and Spider Web Library: A Sample of Student Work, CERL, October, 1975.
- Dugdale, S., and Kibbey, D., Elementary Mathematics with PLATO, CERL, August 1976, second edition July 1977.
- Davis, R., Dugdale, S., Kibbey, D., and Weaver, C., "Representing Knowledge About Mathematics for Computer-Aided Teaching: Part II -- The Diversity of Roles that a Computer Can Play in Assisting Learning," in Machine Representations of Knowledge (eds. E. W. Elcock and D. Michie). Dordrecht: D. Reidel Publishing Company, 1977.
- Dugdale, S., and Vogel, P., "PLATO Instruction for the Hearing Impaired," Proceedings of the Second International Learning Technology Congress and Exposition of the Society for Applied Learning Technology, Orlando, Florida, February, 1978.
- Dugdale, S., and Vogel, P., "Computer-based Instruction for Hearing Impaired Children in the Classroom", American Annals of the Deaf, October, 1978, pp. 730-743.
- Dugdale, S., "Using the Computer to Foster Creative Interaction Among Students", Proceedings of the Association for Educational Data Systems Seventeenth Annual Convention, Detroit, Michigan, May, 1979. Also published as CERL Report E-9, Computer-based Education Research Laboratory, University of Illinois, Urbana.

Appendix V

Index of Lesson Descriptions

Addition by Using an Easier Problem: Part 1, J4
 Addition by Using an Easier Problem: Part 2, J5
 Addition by Using an Easier Problem: Part 3, J6
 Addition by Using an Easier Problem: Part 3 Practice, J7
 Addition by Using an Easier Problem: Part 4, J8
 Addition by Using an Easier Problem: Part 4 Practice, J9
 Addition by Using an Easier Problem: Review, J10
 Addition and Subtraction Drill, I12
 Addition and Subtraction Drill, J11
 Addition and Subtraction of Mixed Numbers with Like Denominators, I4
 Addition and Subtraction Practice with Equivalence Sets, L4
 Addition and Subtraction Practice with Simplifying and Borrowing, I9
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